

Vector Analysis Theorems

1. The Fundamental Theorem of Calculus for Curves states that if $\vec{r}(t)$ is a nice curve in $\mathbf{R}^n$ and f is a nice function in $\mathbf{R}^n$ then

$$\int_{\substack{A \\ \vec{r}}}^{B} \vec{\nabla} f \cdot \overrightarrow{ds} = f(B) - f(A)$$

2. Green's Theorem states that if R is a nice region in $\mathbf{R}^2$ and ∂R is its boundary curve traversed counterclockwise and P and Q are a nice functions on R then

$$\iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx\,dy = \oint_{\partial R} P\,dx + Q\,dy$$

3. Stokes' Theorem states that if S is a nice surface in $\mathbf{R}^3$ and ∂S is its boundary curve traversed counterclockwise as seen from the tip of the normal to S and $\vec{F}$ is a nice vector field on S then

$$\iint_S \vec{\nabla} \times \vec{F} \cdot d\vec{S} = \oint_{\partial S} \vec{F} \cdot d\vec{s}$$

4. Gauss' Theorem states that if V is a volume in $\mathbf{R}^3$ and ∂V is its boundary surface oriented outward from V and $\vec{F}$ is a nice vector field on V then

$$\iiint_V \vec{\nabla} \cdot \vec{F}\,dV = \iint_{\partial V} \vec{F} \cdot \overrightarrow{dS}$$