Name____

Math 311 Exam 1 Spring 2013

Section 501 P. Yasskin

1	/25	4	/10
2	/20	5	/15
3	/30	Total	/100

1. (25 points) Let $A = \begin{pmatrix} 1 & 3 & 0 & 2 & 2 \\ 2 & 7 & 2 & 5 & 5 \\ 0 & 1 & 2 & 2 & 1 \\ 0 & 3 & 6 & 4 & 3 \end{pmatrix}$, $\vec{x} = \begin{pmatrix} p \\ q \\ x \\ y \\ z \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 2 \\ 6 \\ 3 \\ 8 \end{pmatrix}$, $\vec{c} = \begin{pmatrix} 2 \\ 6 \\ 3 \\ 7 \end{pmatrix}$.

Solve both equations $A\vec{x} = \vec{b}$ and $A\vec{x} = \vec{c}$. Give all solutions or say why there are no solutions.

2. (20 points) Let $A = \begin{pmatrix} 1 & 3 & 0 & 2 & 2 \\ 2 & 7 & 2 & 5 & 5 \\ 0 & 1 & 2 & 2 & 1 \\ 0 & 3 & 6 & 4 & 3 \end{pmatrix}$ as in problem 1.

a. Find a basis for $N(A)$, the null space of A. What is $\dim N(A)$, the nullity of A?

b. Find a basis for $R(A)$, the row space of A. What is $\dim R(A)$, the row rank of A?

c. Find a basis for $C(A)$, the column space of A. What is $\dim C(A)$, the column rank of A?

d. Give 2 relations between the 3 numbers $\dim N(A)$, $\dim R(A)$ and $\dim C(A)$ which would be true for any 4×5 matrix A. (No proof.)

3. (30 points) Let $A \in M(n,n)$ and $\vec{b} \in \mathbb{R}^n$. For each of the following conditions, say whether A is singular or non-singular (circle one). Then give a reason. For one and only one of these, you may say this is the definition of singular or non-singular. For the rest, your reason must say why the given condition is equivalent to the definition.

a. A is row reducable to the unit matrix. singular non-singular
Reason:

b. The reduced row echelon form of A has a row of zeros at the bottom. singular non-singular
Reason:

c. A is invertible. singular non-singular
Reason:

d. The equation $A\vec{x} = \vec{0}$ has an infinite number of solutions. singular non-singular
Reason:

e. The equation $A\vec{x} = \vec{b}$ has a unique solution. singular non-singular
Reason:

f. $\det(A) = 0$ singular non-singular
Reason:

4. $\left(10 \text{ points}\right)$ Recall the definitions:

If A is a $p \times q$ matrix then A^{T} is the $q \times p$ matrix whose ij entry is $\left(A^{\mathsf{T}}\right)_{ij} = A_{ji}$.

If A is a $p \times q$ matrix and B is a $q \times r$ matrix then AB is the $p \times r$ matrix whose ij entry is

$(AB)_{ij} = \sum_{k=0}^{q} A_{ik}B_{kj}$.

Use only these definitions to prove $(AB)^{\mathsf{T}} = B^{\mathsf{T}}A^{\mathsf{T}}$.

5. $\left(15 \text{ points}\right)$ In the vector space $V = \mathbb{R}^{+}$ with $a \oplus b = ab$ and $p \odot a = a^{p}$, write out each of the following facts as identities about ordinary multiplication and exponentiation.

a. $p \odot (a \oplus b) = (p \odot a) \oplus (p \odot b)$

b. $(p + q) \odot a = (p \odot a) \oplus (q \odot a)$

c. $(pq) \odot a = p \odot (q \odot a)$

d. $0 \otimes a = \vec{0}$

e. $p \otimes \vec{0} = \vec{0}$