1	/40
2	/30
3	/40
Total	/110

MATH 311 Section 501 Exam 2 Spring 2013 P. Yasskin

1. (35 points+5 e.c.) Consider the vector space $M(2,2)$ of 2×2 real matrices. Consider the bases

$$E_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad E_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad E_3 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \quad E_4 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$F_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad F_2 = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \quad F_3 = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \quad F_4 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

Consider the function $L : M(2,2) \to M(2,2)$ given by

$$L(X) = 2X - X^{\mathsf{T}}$$

a. (5 pts) Show L is linear.

b. (5 pts) Find the matrix of L relative to the E-basis. Call it $\underset{E \leftarrow E}{A}$.

HINT: A is NOT a 2×2 matrix!

c. (5 pts) Find the change of basis matrix $\underset{E \leftarrow F}{C}$ from the F-basis to the E-basis.

d. (5 pts) Find the change of basis matrix $\underset{F \leftarrow E}{C}$ from the E-basis to the F-basis.

e. (10 pts) Find the matrix of L relative to the F-basis. Call it $\underset{F \leftarrow F}{B}$.

f. (5 pts) Find $\underset{F \leftarrow F}{B}$ by a second method.

g. (5 pts Extra Credit) Looking at your solutions to (b), (e) and (f), find 3 linearly independent eigenvectors of L. What are their eigenvalues?

2. (30 points) Consider the vector space $M(2,2)$ of 2×2 real matrices.
Consider the function of two matrices

$$\langle X, Y \rangle = \mathrm{tr}\left(XDY^{\mathsf{T}}\right) \quad \text{where} \quad D = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$$

where tr means trace (sum of diagonal elements) and $^{\mathsf{T}}$ means transpose.

a. (10 pts) Show $\langle X, Y \rangle$ is an inner product.

HINT: First compute the inner product of $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $Y = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$.

b. (10 pts) Find the angle between the matrices $P = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $Q = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

c. (10 pts) Let $V = \mathrm{Span}\{P, Q\}$ where P and Q are given in (b).

- Find $V^{\perp}$ the orthogonal subspace to V within $M(2,2)$.

- Find a basis for $V^{\perp}$ and its dimension.

3. (35 points+5 e.c.) Consider the matrix $A = \begin{pmatrix} 0 & 6 \\ -1 & 5 \end{pmatrix}$.

a. (15 pts) Find the eigenvalues and corresponding eigenvectors of A.

b. (10 pts) Find the diagonalizing matrix X so that $A = XDX^{-1}$ where D is diagonal. What are X^{-1} and D? Just state them. No derivation.

$X = \begin{pmatrix} & \\ & \end{pmatrix}$ $X^{-1} = \begin{pmatrix} & \\ & \end{pmatrix}$ $D = \begin{pmatrix} & \\ & \end{pmatrix}$

c. (5 pts) Find A^4.

d. (5 pts) Find $\cos(\pi A)$.

e. (5 pts Extra Credit) What are the eigenvalues and corresponding eigenvectors of $5A$?