MATH 311 Section 501 Quiz on Chain Rule Spring 2013 P. Yasskin

Name:_____

Definition: $x = x(u,v)$ and $y = y(u,v)$, then the Jacobian matrix of x and y with respect to u and v is

$$\frac{D(x,y)}{D(u,v)} = \begin{pmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{pmatrix}$$

and similarly with more or less variables.

1. Duke Skywater is flying the Millenium Eagle through a region of intergalactic space containing a deadly polaron field and a life giving axion field. If the polaron density is P and the axion density is A, then the combined danger strength is $F = 9\frac{P}{A}$. At galactic time $t = 47241.3$, Duke is located at the point $(x,y,z) = (47,23,21)$ lightyears and has velocity $\vec{v} = (.2,-.1,.3)$ lightyears/year. At that instant, he measures the polaron density is $P = 5\times10^{40}$ polarons/millilightyear^3 and has gradient $\vec{\nabla}P = (-4\times10^{38}, 2\times10^{38}, 3\times10^{38})$ polarons/millilightyear^4 and the axion density is $A = 3\times10^{30}$ axions/millilightyear^3 and has gradient $\vec{\nabla}A = (5\times10^{28}, -3\times10^{28}, 1\times10^{28})$ axions/millilightyear^4. What is the danger strength F and its time rate of change $\frac{dF}{dt}$ at $t = 47241.3$? Is the danger increasing or decreasing? Follow these steps: (If you don't have a formula for some function, just write the name of the derivative, e.g. $\frac{\partial P}{\partial y}$.)

a. Find the danger strength F at $t = 47241.3$

b. Find the Jacobian matrix $\frac{D(F)}{D(P,A)}$ in general and then at $t = 47241.3$.

c. Find the Jacobian matrix $\frac{D(P,A)}{D(x,y,z)}$ in general and then at $t = 47241.3$.

d. Find the Jacobian matrix $\frac{D(x,y,z)}{D(t)}$ in general and then at $t = 47241.3$.

e. Find the Jacobian matrix $\frac{D(F)}{D(t)} = \left(\frac{dF}{dt}\right)$ at $t = 47241.3$.

f. Is the danger increasing or decreasing?