

Name _____

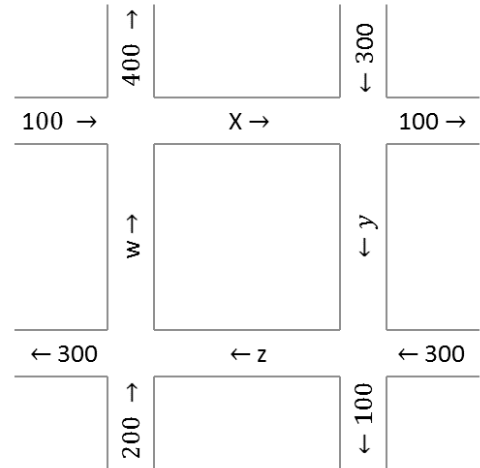
Math 311 Exam 1 Version A Spring 2015

Section 503 P. Yasskin

Points indicated. Show all work.

1	/30	5	/20
2	/10	6	/15
3	/10	7	/15
4	/10	Total	/110

1. (30 points) Solve the traffic flow system shown at the right. Find the smallest non-negative values of w , x , y , z . In your augmented matrix, keep the variables in the order w , x , y , z .



2. (10 points) By definition, a matrix, A , is nilpotent with degree 2 if $A^2 = \mathbf{0}$.
Prove if A is nilpotent with degree 2, then $\mathbf{1} + A$ is non-singular and $(\mathbf{1} + A)^{-1} = \mathbf{1} - A$.

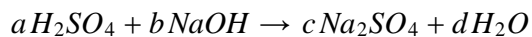
3. (10 points) For an $n \times n$ matrix A , define its trace to be $tr(A) = \sum_{i=1}^n A_{ii}$ i.e. the sum of its diagonal entries. Prove, for $n \times n$ matrices A and B , $tr(AB) = tr(BA)$.

4. (10 points) A matrix A satisfies $E_1 E_2 E_3 A = B$ where

$$E_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad E_2 = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad E_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix} \quad B = \begin{pmatrix} -3 & * & 5 \\ 0 & 4 & * \\ 0 & 0 & 2 \end{pmatrix}$$

and the *'s represent unknown non-zero numbers. Find $\det A$.

5. (20 points) Sulfuric acid (H_2SO_4) combines with sodium hydroxide ($NaOH$) to produce sodium sulfate (Na_2SO_4) and water (H_2O) according to the chemical equation:



To balance this chemical equation, you must solve the system

$$H : \quad 2a + b = 2d$$

$$S : \quad a = c$$

$$O : \quad 4a + b = 4c + d$$

$$Na : \quad b = 2c$$

- a. Write out the augmented matrix for this system of 4 equations in 4 unknowns. DO NOT SOLVE.

- b. Compute the determinant of the matrix of coefficients.

- c. What property of the augmented matrix says there is at least one solution? (Circle one.)

- i. The fact that the determinant of the matrix of coefficients is zero.
- ii. The fact that the determinant of the matrix of coefficients is non-zero.
- iii. The fact that the matrix of coefficients is square (4×4).
- iv. The fact that the system is homogeneous (the right hand sides are all zero).

- d. What additional property says there are infinitely many solutions? (Circle one.)

- i. The fact that the determinant of the matrix of coefficients is zero.
- ii. The fact that the determinant of the matrix of coefficients is non-zero.
- iii. The fact that the matrix of coefficients is square (4×4).
- iv. The fact that the system is homogeneous (the right hand sides are all zero).

6. (15 points) Let $A = \begin{pmatrix} a & 4 & 3 \\ b & 1 & 2 \\ 0 & c & d \end{pmatrix}$. Given that $\det(A) = 3$, determine each of the following:

$$\begin{vmatrix} a & 4 & 3 \\ 0 & c & d \\ b & 1 & 2 \end{vmatrix} = \underline{\hspace{2cm}} \qquad \begin{vmatrix} a & 4 & 3 \\ b & 1+c & 2+d \\ 0 & c & d \end{vmatrix} = \underline{\hspace{2cm}} \qquad \begin{vmatrix} a & 8 & 3 \\ b & 2 & 2 \\ 0 & 2c & d \end{vmatrix} = \underline{\hspace{2cm}}$$

$$\det(2A) = \underline{\hspace{2cm}} \qquad \det(A^{-1}) = \underline{\hspace{2cm}}$$

7. (15 points) For each of the following sets with operations, determine whether it forms a vector space. If it does, just say "Yes". If it does not, say "No" and give an axiom or other property it violates and show why.

- a. The set of all power series centered at 2, $S = \left\{ a = \sum_{n=0}^{\infty} a_n(x-2)^n \right\}$ with

$$a \oplus b = \sum_{n=0}^{\infty} (a_n + b_n)(x-2)^n \text{ and } \alpha \odot a = \sum_{n=0}^{\infty} \alpha a_n(x-2)^n$$

- b. $F_{\text{odd}}[-1, 1] = \{f : [-1, 1] \rightarrow \mathbb{R} \mid f(-x) = -f(x)\}$ with
 $(f \oplus g)(x) = f(-x) + g(-x)$ and $(\alpha \odot f)(x) = \alpha f(-x)$

- c. $F_{\text{even}}[-1, 1] = \{f : [-1, 1] \rightarrow \mathbb{R} \mid f(-x) = f(x)\}$ with
 $(f \oplus g)(x) = f(-x) + g(-x)$ and $(\alpha \odot f)(x) = \alpha f(-x)$