Name____

Math 311 Exam 3 Version A Spring 2015

Section 502 P. Yasskin

Points indicated. Show all work.

1	/20	3	/30
2	/36	4	/26
		Total	/112

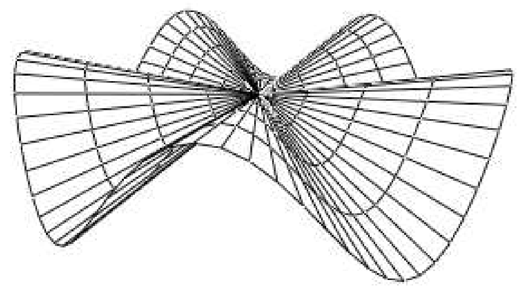
1. (20 points) Compute $\iint_S \vec{\nabla} \times \vec{F} \cdot d\vec{S}$ for $\vec{F} = (-y, x, z)$ over the "clam shell" surface, S, parametrized by

$$\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r\sin(5\theta))$$

for $r \le 3$ oriented upward.

HINTS: Use Stokes Theorem.

What is the value of r on the boundary?



2. (36 points) Let $V = Span(e^{2x} + e^{-2x}, e^{2x} - e^{-2x})$ be the vector space of functions spanned by the basis

$$e_1 = e^{2x} + e^{-2x}, \qquad e_2 = e^{2x} - e^{-2x}$$

Consider the linear operator $L : V \to V$ given by $L(f) = 3\frac{df}{dx}$. Our goals are to compute the eigenvalues and eigenfunctions of the linear operator L, to find the similarity transformation which diagonalizes the matrix of L and use this similarity transformation to compute a matrix power.

a. (5 pts) Find the matrix of L relative to the (e_1, e_2) basis. Call it $\underset{e \leftarrow e}{A}$.

b. (3 pts) Find the characteristic polynomial for $\underset{e \leftarrow e}{A}$.

Factor it and identify the eigenvalues of $\underset{e \leftarrow e}{A}$. These are also the eigenvalues of L.

c. (8 pts) Find the eigenvector(s) of $\underset{e \leftarrow e}{A}$ for each eigenvalue, as vectors in $\mathbb{R}^2$.

Name them $\vec{v}_1$ and $\vec{v}_2$.

d. (6 pts) Convert the eigenvectors of $\underset{e \leftarrow e}{A}$ into eigenfunctions of L as functions in V.

Name them f_1 and f_2 and simplify them.

Then compute $L(f_1)$ and $L(f_2)$ to verify f_1 and f_2 are eigenfunctions.

Hint: Remember that the components of $\vec{v}_1$ and $\vec{v}_2$ are components of f_1 and f_2 relative to the (e_1, e_2) basis.

e. (3 pts) Using the eigenfunctions as a new (f_1, f_2) basis for V, find the matrix of L relative to the (f_1, f_2) basis. Call it $\underset{f\leftarrow f}{D}$.

f. (5 pts) Find the change of basis matrices $\underset{e\leftarrow f}{C}$ and $\underset{f\leftarrow e}{C}$ between the (e_1, e_2) basis to the (f_1, f_2) bases. Be sure to identify which is which.

g. (2 pts) A and D are related by a similarity transformation $A = S^{-1}DS$.
Identify S as $\underset{e\leftarrow f}{C}$ or $\underset{f\leftarrow e}{C}$.

h. (4 pts) Compute A^{12} and A^{25}.

3. (30 points) The density, ρ, of an ideal gas is related to its pressure, P, and its absolute temperature, T, by the equation $\rho = \frac{P}{kT}$ where k is a constant which depends on the particular ideal gas. We are considering an ideal gas for which $k = 10^{-4}$ atm•m^3/kg/°K. At the current time, $t = t_0$, a flying robotic nanobot is located at $(x, y, z) = (2, 1, 3)^{\mathsf{T}}$ m and has velocity $\vec{v} = (.4, .5, .2)^{\mathsf{T}}$ m/sec. The nanobot measures the current pressure is $P = 2$ atm while its gradient is $\vec{\nabla} P = (-.03, .01, .02)$atm/m. Similarly, the nanobot measures the current temperature is $T = 250$ °K while its gradient is $\vec{\nabla} T = (3, -2, -4)$ °K/m.

a. (2 pts) Find the current density, ρ.

b. (6 pts) Find the Jacobian matrix of the density $\frac{D(\rho)}{D(P,T)}$ in general (in terms of symbols like $\frac{\partial \rho}{\partial T}$), then in terms of P and T, and finally at the current time $t = t_0$.

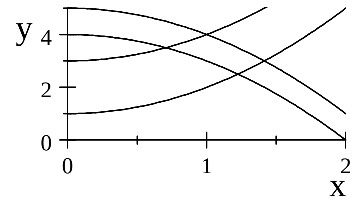
c. (4 pts) Find the Jacobian matrix $\frac{D(P,T)}{D(x,y,z)}$ in general (in terms of symbols like $\frac{\partial P}{\partial y}$) and then at the current time $t = t_0$.

d. (4 pts) Find the Jacobian matrix $\frac{D(x,y,z)}{D(t)}$ in general and then at $t = t_0$.

e. (6 pts) Find the time rate of change of the pressure as seen by the nanobot, at the current time $t = t_0$. Is the pressure currently increasing or decreasing?

f. (8 pts) Find the time rate of change of the density as seen by the nanobot, at the current time $t = t_0$. Is the density currently increasing or decreasing?

4. (26 points) Compute the integral $\iint x\,dA$ over the region in the first quadrant bounded by
$y = 1 + x^2$, $y = 3 + x^2$, $y = 4 - x^2$, and $y = 5 - x^2$.



a. (4 pts) Define the curvilinear coordinates u and v by $y = u + x^2$ and $y = v - x^2$.
What are the 4 boundaries in terms of u and v?

b. (4 pts) Solve for x and y in terms of u and v. Express the results as a position vector.

c. (4 pts) Find the coordinate tangent vectors:

d. (8 pts) Compute the Jacobian factor:

e. (6 pts) Compute the integral: