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Math 311 Exam 3 Version B Spring 2015

Section 502 P. Yasskin

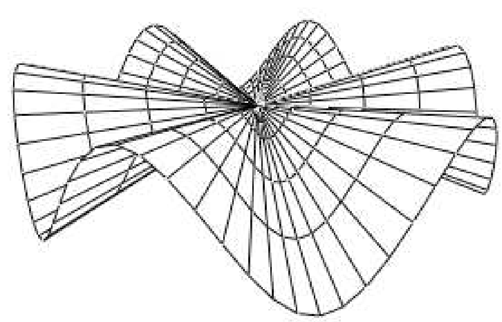
Points indicated. Show all work.

1	/20	3	/30
2	/36	4	/26
		Total	/112

1. (20 points) Compute $\iint_S \vec\nabla \times \vec F \cdot d\vec S$ for $\vec F = (-y, x, z)$ over the "clam shell" surface, S, parametrized by

$$\vec R(r,\theta) = (r\cos\theta, r\sin\theta, r\sin(6\theta))$$

for $r \le 2$ oriented upward.



HINTS: Use Stokes Theorem.

What is the value of r on the boundary?

2. (36 points) Let $V = Span(e^{2x} + e^{-2x}, e^{2x} - e^{-2x})$ be the vector space of functions spanned by the basis

$$e_1 = e^{2x} + e^{-2x}, \qquad e_2 = e^{2x} - e^{-2x}$$

Consider the linear operator $L : V \to V$ given by $L(f) = 4\frac{df}{dx}$. Our goals are to compute the eigenvalues and eigenfunctions of the linear operator L, to find the similarity transformation which diagonalizes the matrix of L and use this similarity transformation to compute a matrix power.

a. (5 pts) Find the matrix of L relative to the (e_1, e_2) basis. Call it $\underset{e\leftarrow e}{A}$.

b. (3 pts) Find the characteristic polynomial for $\underset{e\leftarrow e}{A}$.
Factor it and identify the eigenvalues of $\underset{e\leftarrow e}{A}$. These are also the eigenvalues of L.

c. (8 pts) Find the eigenvector(s) of $\underset{e\leftarrow e}{A}$ for each eigenvalue, as vectors in $\mathbb{R}^2$.
Name them $\vec{v}_1$ and $\vec{v}_2$.

d. (6 pts) Convert the eigenvectors of $\underset{e\leftarrow e}{A}$ into eigenfunctions of L as functions in V.
Name them f_1 and f_2 and simplify them.
Then compute $L(f_1)$ and $L(f_2)$ to verify f_1 and f_2 are eigenfunctions.

Hint: Remember that the components of $\vec{v}_1$ and $\vec{v}_2$ are components of f_1 and f_2 relative to the (e_1, e_2) basis.

e. (3 pts) Using the eigenfunctions as a new (f_1, f_2) basis for V, find the matrix of L relative to the (f_1, f_2) basis. Call it $\underset{f\leftarrow f}{D}$.

f. (5 pts) Find the change of basis matrices $\underset{e\leftarrow f}{C}$ and $\underset{f\leftarrow e}{C}$ between the (e_1, e_2) basis to the (f_1, f_2) bases. Be sure to identify which is which.

g. (2 pts) A and D are related by a similarity transformation $A = S^{-1}DS$. Identify S as $\underset{e\leftarrow f}{C}$ or $\underset{f\leftarrow e}{C}$.

h. (4 pts) Compute A^{10} and A^{15}.

3. $\left(30\text{ points}\right)$ The density, ρ, of an ideal gas is related to its pressure, P, and its absolute temperature, T, by the equation $\rho = \frac{P}{kT}$ where k is a constant which depends on the particular ideal gas. We are considering an ideal gas for which $k = 10^{-4}$ atm•m^3/kg/°K. At the current time, $t = t_0$, a flying robotic nanobot is located at $(x, y, z) = (2, 1, 3)^\intercal$ m and has velocity $\vec{v} = (.4, .5, .2)^\intercal$ m/sec. The nanobot measures the current pressure is $P = 2$ atm while its gradient is $\vec{\nabla}P = (-.06, .02, .04)$atm/m. Similarly, the nanobot measures the current temperature is $T = 250$ °K while its gradient is $\vec{\nabla}T = (3, -2, -4)$ °K/m.

a. $\left(2\text{ pts}\right)$ Find the current density, ρ.

b. $\left(6\text{ pts}\right)$ Find the Jacobian matrix of the density $\frac{D(\rho)}{D(P,T)}$ in general (in terms of symbols like $\frac{\partial \rho}{\partial T}$), then in terms of P and T, and finally at the current time $t = t_0$.

c. $\left(4\text{ pts}\right)$ Find the Jacobian matrix $\frac{D(P,T)}{D(x,y,z)}$ in general (in terms of symbols like $\frac{\partial P}{\partial y}$) and then at the current time $t = t_0$.

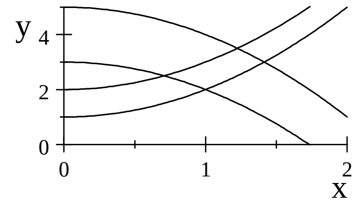
d. $\left(4\text{ pts}\right)$ Find the Jacobian matrix $\frac{D(x,y,z)}{D(t)}$ in general and then at $t = t_0$.

e. $\left(6 \text{ pts}\right)$ Find the time rate of change of the pressure as seen by the nanobot, at the current time $t = t_0$. Is the pressure currently increasing or decreasing?

f. $\left(8 \text{ pts}\right)$ Find the time rate of change of the density as seen by the nanobot, at the current time $t = t_0$. Is the density currently increasing or decreasing?

4. $\left(26 \text{ points}\right)$ Compute the integral $\iint x\,dA$ over the region in the first quadrant bounded by

$y = 1 + x^2$, $y = 2 + x^2$, $y = 3 - x^2$, and $y = 5 - x^2$.



a. $\left(4 \text{ pts}\right)$ Define the curvilinear coordinates u and v by $y = u + x^2$ and $y = v - x^2$. What are the 4 boundaries in terms of u and v?

b. $\left(4 \text{ pts}\right)$ Solve for x and y in terms of u and v. Express the results as a position vector.

c. $\left(4 \text{ pts}\right)$ Find the coordinate tangent vectors:

d. $\left(8 \text{ pts}\right)$ Compute the Jacobian factor:

e. $\left(6 \text{ pts}\right)$ Compute the integral: