

3. Consider the vector space $M(2,2)$ of 2×2 matrices with the basis

$$E_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad E_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad E_3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad E_4 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Which of the following are the components of the matrix $A = \begin{pmatrix} 9 & 5 \\ 1 & 1 \end{pmatrix}$ relative to the E basis?

- a. $(A)_E = \begin{pmatrix} 2 & 1 & 4 & 3 \end{pmatrix}^\top$
- b. $(A)_E = \begin{pmatrix} 5 & 4 & 3 & 2 \end{pmatrix}^\top$
- c. $(A)_E = \begin{pmatrix} 4 & 5 & 2 & 3 \end{pmatrix}^\top$
- d. $(A)_E = \begin{pmatrix} 1 & 2 & 3 & 4 \end{pmatrix}^\top$
- e. $(A)_E = \begin{pmatrix} 5 & -4 & 3 & -2 \end{pmatrix}^\top$

4. Consider the vector space $C_2([0,1])$ of real valued function on the interval $[0,1]$ whose second derivatives exist and are continuous with the inner product $\langle f, g \rangle = \int_{-1}^1 f(x)g(x)dx$. Consider the subspace $V = \text{Span}(x, x^2)$ spanned by the basis $v_1 = x$, $v_2 = x^2$. Which of the following is an orthonormal basis for V ?

- a. $u_1 = x \quad u_2 = x^2$
- b. $u_1 = \sqrt{\frac{5}{2}}x \quad u_2 = \sqrt{\frac{7}{2}}x^2$
- c. $u_1 = x \quad u_2 = x^2 - \frac{3}{4}x$
- d. $u_1 = \sqrt{\frac{3}{2}}x \quad u_2 = \sqrt{\frac{5}{2}}x^2$
- e. $u_1 = \sqrt{2}x \quad u_2 = 4\sqrt{5}x^2 - 3\sqrt{5}x$

5. Consider the second derivative linear operator $L : P_6 \rightarrow P_6 : L(p) = \frac{d^2 p}{dx^2}$ on the space of polynomials of degree less than 6. Find the image, $\text{Im}(L)$.

HINT: Let $p = a + bx + cx^2 + dx^3 + ex^4 + fx^5$.

- $\text{Im}(L) = \text{Span}(1)$
- $\text{Im}(L) = \text{Span}(1, x)$
- $\text{Im}(L) = \text{Span}(1, x, x^2, x^3)$
- $\text{Im}(L) = \text{Span}(x^2, x^3, x^4, x^5)$
- $\text{Im}(L) = \text{Span}(x, x^2, x^3, x^4, x^5)$

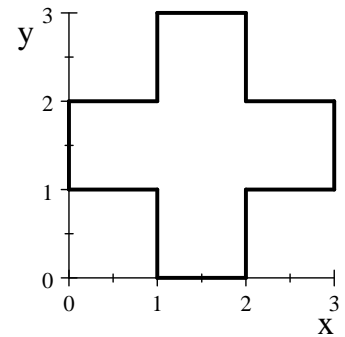
6. Consider the second derivative linear operator $L : P_6 \rightarrow P_6 : L(p) = \frac{d^2 p}{dx^2}$ on the space of polynomials of degree less than 6. Find the kernel, $\text{Ker}(L)$.

HINT: Let $p = a + bx + cx^2 + dx^3 + ex^4 + fx^5$.

- $\text{Ker}(L) = \text{Span}(1)$
- $\text{Ker}(L) = \text{Span}(1, x)$
- $\text{Ker}(L) = \text{Span}(1, x, x^2)$
- $\text{Ker}(L) = \text{Span}(x^2, x^3, x^4, x^5)$
- $\text{Ker}(L) = \text{Span}(x, x^2, x^3, x^4, x^5)$

7. Compute the line integral $\oint \vec{F} \cdot d\vec{s}$ clockwise around the complete boundary of the plus sign, shown at the right, for the vector field $\vec{F} = (4x^3 + 2y, 4y^3 - 3x)$.

- 25
- 5
- 0
- 5
- 25



8. Consider the vector space $M(2,2)$ of 2×2 matrices. Let $A = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$. Consider the linear function, $L : M(2,2) \rightarrow M(2,2) : L(X) = AX - XA$. Which of the following is not an eigenvalue and corresponding eigenmatrix (eigenvector) of L ?

HINT: Let $X = \begin{pmatrix} x & y \\ z & w \end{pmatrix}$.

- a. $\lambda = 2$ $X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
- b. $\lambda = 1$ $X = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$
- c. $\lambda = 0$ $X = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$
- d. $\lambda = 0$ $X = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
- e. $\lambda = -1$ $X = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

9. (16 points) Which of the following is an inner product on \mathbb{R}^2 ? If not, why not?

Put \times 's in the correct boxes. No part credit.

Let $\vec{x} = (x_1, x_2)$, $\vec{y} = (y_1, y_2)$.

	$\langle \vec{x}, \vec{y} \rangle =$	Inner Product?		Why not?			
		Yes	No	Not	Not	Not	Positive but Not
				Symmetric	Linear	Positive	Positive Definite
a.	$x_1y_1 + 2x_2y_2$						
b.	$x_1y_1 + 2x_1y_2 + 2x_2y_2$						
c.	$x_1^2y_1^2 + 2x_2^2y_2^2$						
d.	$x_1y_1 + x_1y_2 + x_2y_1 + 2x_2y_2$						
e.	$x_1y_1 - x_1y_2 - x_2y_1 + 2x_2y_2$						
f.	$x_1y_1 - x_1y_2 - x_2y_1 + x_2y_2$						
g.	$x_1y_1 + x_1y_2 + x_2y_1 + x_2y_2$						
h.	$x_1y_1 - x_2y_2$						

10. (20 points) Let $M(2,3)$ be the vector space of 2×3 matrices.

Consider the subspace $V = \text{Span}(A_1, A_2, A_3, A_4)$ where

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 3 & -2 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 2 & 2 & 0 \\ 4 & 6 & -4 \end{pmatrix}, \quad A_3 = \begin{pmatrix} -1 & 0 & 2 \\ -3 & 0 & 0 \end{pmatrix}, \quad A_4 = \begin{pmatrix} -1 & 2 & 6 \\ -5 & 6 & -4 \end{pmatrix}$$

Find a basis for V . What is the $\dim V$?

11. (28 points) Compute $\iint_H \vec{F} \cdot d\vec{S}$ over the hemisphere $z = \sqrt{25 - x^2 - y^2}$ oriented upward, for the vector field $\vec{F} = (x^3 - 4y^2 - 4z^2, -4x^2 + y^3 - 4z^2, -4x^2 - 4y^2 + z^3)$.
HINT: Use Gauss' Theorem by following these steps:

a. Write out Gauss' Theorem for the Volume, V , which is the solid hemisphere $0 \leq z \leq \sqrt{25 - x^2 - y^2}$. Split up the boundary, ∂V , into two pieces, the hemisphere, H , and the disk, D , at the bottom. State orientations. Solve for the integral you want.

b. Compute the volume integral using spherical coordinates.

(continued)

c. Compute the other surface integral over D by parametrizing the disk, computing the tangent vectors and normal vector, checking the orientation, evaluating the vector field on the disk and doing the integral.

d. Solve for the original integral.