Name____________________________________

Math 311 Final Exam Version A Spring 2015

Section 503 P. Yasskin

Multiple Choice: 5 points each. No Part Credit

Work Out: Points indicated. Show all work.

| | | | |
|---|---|---|---|
| 1-8 | /40 | 10 | /20 |
| 9 | /16 | 11 | /28 |
| | | Total | /104 |

**1**. Hydrocloric acid $(HCl)$ and sodium hydroxide $(NaOH)$ react to produce sodium chloride $(NaCl)$ and water $(H_2O)$ according to the chemical equation:

$$aHCl + bNaOH \to cNaCl + dH_2O$$

Which of the following is the augmented matrix which is used to solve this chemical equation? (Put the elements in the order $H$, $Cl$, $Na$, $O$.)

**a**. $\left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & -1 & -1 & 0 & 0 \\ -2 & 0 & 0 & -1 & 0 \end{array}\right)$ **b**. $\left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 \end{array}\right)$

**c**. $\left(\begin{array}{cccc|c} 1 & 1 & 0 & -2 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{array}\right)$ **d**. $\left(\begin{array}{cccc|c} 1 & 1 & 0 & 2 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{array}\right)$

**2**. Suppose $A$ is nilpotent, i.e. $A^2 = 0$. Which of the following is true?

**a**. $A + \mathbf{1}$ is invertible and $(A + \mathbf{1})^{-1} = A - \mathbf{1}$
**b**. $A + \mathbf{1}$ is invertible and $(A + \mathbf{1})^{-1} = \mathbf{1} - A$
**c**. $A + \mathbf{1}$ is invertible and $(A + \mathbf{1})^{-1} = \mathbf{1} - \mathbf{2A}$
**d**. $A + \mathbf{1}$ is invertible and $(A + \mathbf{1})^{-1} = \mathbf{1} + A$
**e**. $A + \mathbf{1}$ is not invertible

**3.** Consider the vector space $M(2,2)$ of $2 \times 2$ matrices with the basis

$$E_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad E_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad E_3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad E_4 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Which of the following are the components of the matrix $A = \begin{pmatrix} 9 & 5 \\ 1 & 1 \end{pmatrix}$ relative to the $E$ basis?

**a.** $(A)_E = \begin{pmatrix} 2 & 1 & 4 & 3 \end{pmatrix}^T$
**b.** $(A)_E = \begin{pmatrix} 1 & 2 & 3 & 4 \end{pmatrix}^T$
**c.** $(A)_E = \begin{pmatrix} 4 & 5 & 2 & 3 \end{pmatrix}^T$
**d.** $(A)_E = \begin{pmatrix} 5 & 4 & 3 & 2 \end{pmatrix}^T$
**e.** $(A)_E = \begin{pmatrix} 5 & -4 & 3 & -2 \end{pmatrix}^T$

**4.** Consider the vector space $C_2([0,1]$ of real valued function on the interval $[0,1]$ whose second derivatives exist and are continuous with the inner product $\langle f,g \rangle = \int_{-1}^{1} f(x)\,g(x)\,dx$. Consider the subspace $V = Span(x, x^2)$ spanned by the basis $v_1 = x$, $v_2 = x^2$. Which of the following is an orthonormal basis for $V$?

**a.** $u_1 = x \qquad u_2 = x^2$
**b.** $u_1 = x \qquad u_2 = x^2 - \frac{3}{4}x$
**c.** $u_1 = \sqrt{\frac{3}{2}}\,x \qquad u_2 = \sqrt{\frac{5}{2}}\,x^2$
**d.** $u_1 = \sqrt{\frac{5}{2}}\,x \qquad u_2 = \sqrt{\frac{7}{2}}\,x^2$
**e.** $u_1 = \sqrt{2}\,x \qquad u_2 = 4\sqrt{5}\,x^2 - 3\sqrt{5}\,x$

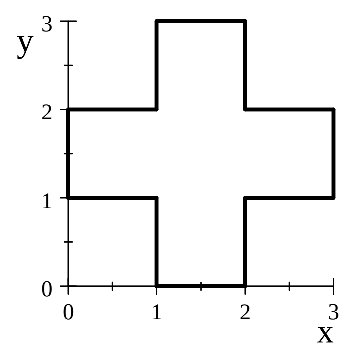
**5**. Consider the second derivative linear operator $L : P_6 \to P_6 : L(p) = \dfrac{d^2p}{dx^2}$ on the space of polynomials of degree less than $6$. Find the kernel, $Ker(L)$.
HINT: Let $p = a + bx + cx^2 + dx^3 + ex^4 + fx^5$.

**a**. $Ker(L) = Span(1)$
**b**. $Ker(L) = Span(1, x)$
**c**. $Ker(L) = Span(1, x, x^2)$
**d**. $Ker(L) = Span(x^2, x^3, x^4, x^5)$
**e**. $Ker(L) = Span(x, x^2, x^3, x^4, x^5)$

**6**. Consider the second derivative linear operator $L : P_6 \to P_6 : L(p) = \dfrac{d^2p}{dx^2}$ on the space of polynomials of degree less than $6$. Find the image, $\mathrm{Im}(L)$.
HINT: Let $p = a + bx + cx^2 + dx^3 + ex^4 + fx^5$.

**a**. $\mathrm{Im}(L) = Span(1)$
**b**. $\mathrm{Im}(L) = Span(1, x)$
**c**. $\mathrm{Im}(L) = Span(1, x, x^2, x^3)$
**d**. $\mathrm{Im}(L) = Span(x^2, x^3, x^4, x^5)$
**e**. $\mathrm{Im}(L) = Span(x, x^2, x^3, x^4, x^5)$

**7**. Compute the line integral $\oint \vec{F} \cdot d\vec{s}$ clockwise around the complete boundary of the plus sign, shown at the right, for the vector field $\vec{F} = (4x^3 + 2y, 4y^3 - 3x)$.




**a**. 25
**b**. 5
**c**. 0
**d**. $-5$
**e**. $-25$

**8**. Consider the vector space $M(2,2)$ of $2 \times 2$ matrices. Let $A = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$. Consider the linear function, $L : M(2,2) \to M(2,2) : L(X) = AX - XA$. Which of the following is not an eigenvalue and corresponding eigenmatrix (eigenvector) of $L$?

HINT: Let $X = \begin{pmatrix} x & y \\ z & w \end{pmatrix}$.

**a**. $\lambda = -1$ $\quad X = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

**b**. $\lambda = 0$ $\quad X = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

**c**. $\lambda = 0$ $\quad X = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

**d**. $\lambda = 1$ $\quad X = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

**e**. $\lambda = 2$ $\quad X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

**9.** (16 points) Which of the following is an inner product on $\mathbb{R}^2$? If not, why not?
Put $\times$'s in the correct boxes. No part credit.
Let $\vec{x} = (x_1, x_2)$, $\vec{y} = (y_1, y_2)$.

| | | Inner Product? | | Why not? | | | |
|---|---|---|---|---|---|---|---|
| | | | | Not | Not | Not | Positive but Not |
| | $\langle \vec{x}, \vec{y} \rangle =$ | Yes | No | Symmetric | Linear | Positive | Positive Definite |
| **a.** | $x_1y_1 + 2x_2y_2$ | | | | | | |
| **b.** | $x_1^2y_1^2 + 2x_2^2y_2^2$ | | | | | | |
| **c.** | $x_1y_1 + 2x_1y_2 + 2x_2y_2$ | | | | | | |
| **d.** | $x_1y_1 - x_1y_2 - x_2y_1 + x_2y_2$ | | | | | | |
| **e.** | $x_1y_1 + x_1y_2 + x_2y_1 + x_2y_2$ | | | | | | |
| **f.** | $x_1y_1 + x_1y_2 + x_2y_1 + 2x_2y_2$ | | | | | | |
| **g.** | $x_1y_1 - x_1y_2 - x_2y_1 + 2x_2y_2$ | | | | | | |
| **h.** | $x_1y_1 - x_2y_2$ | | | | | | |

**10**. (20 points) Let $M(2,3)$ be the vector space of $2 \times 3$ matrices.

Consider the subspace $V = Span(A_1, A_2, A_3, A_4)$ where

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 3 & -2 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 2 & 2 & 0 \\ 4 & 6 & -4 \end{pmatrix}, \quad A_3 = \begin{pmatrix} -1 & 0 & 2 \\ -3 & 0 & 0 \end{pmatrix}, \quad A_4 = \begin{pmatrix} 1 & 3 & 4 \\ 0 & 9 & -6 \end{pmatrix}$$

Find a basis for $V$. What is the $\dim V$ ?

**11.** (28 points) Compute $\iint\limits_H \vec{F} \cdot d\vec{S}$ over the hemisphere $z = \sqrt{25 - x^2 - y^2}$ oriented upward, for the vector field $\vec{F} = (x^3 + 4y^2 + 4z^2, 4x^2 + y^3 + 4z^2, 4x^2 + 4y^2 + z^3)$.

HINT: Use Gauss' Theorem by following these steps:

**a.** Write out Gauss' Theorem for the Volume, $V$, which is the solid hemisphere $0 \le z \le \sqrt{25 - x^2 - y^2}$. Split up the boundary, $\partial V$, into two pieces, the hemisphere, $H$, and the disk, $D$, at the bottom. State orientations. Solve for the integral you want.

**b.** Compute the volume integral using spherical coordinates.

(continued)

**c**. Compute the other surface integral over $D$ by parametrizing the disk, computing the tangent vectors and normal vector, checking the orientation, evaluating the vector field on the disk and doing the integral.

**d**. Solve for the original integral.