Name____

Math 311 Final Exam Version B Spring 2015
Section 503 P. Yasskin

1-8	/40	10	/20
9	/16	11	/28
		Total	/104

Multiple Choice: 5 points each. No Part Credit
Work Out: Points indicated. Show all work.

1. Hydrocloric acid (HCl) and sodium hydroxide $(NaOH)$ react to produce sodium chloride $(NaCl)$ and water (H_2O) according to the chemical equation:

$$aHCl + bNaOH \rightarrow cNaCl + dH_2O$$

Which of the following is the augmented matrix which is used to solve this chemical equation? (Put the elements in the order H, Cl, Na, O.)

a. $\left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & -1 & -1 & 0 & 0 \\ -2 & 0 & 0 & -1 & 0 \end{array}\right)$ **b.** $\left(\begin{array}{cccc|c} 1 & 1 & 0 & 2 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{array}\right)$

c. $\left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 \end{array}\right)$ **d.** $\left(\begin{array}{cccc|c} 1 & 1 & 0 & -2 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{array}\right)$

2. Suppose A is nilpotent, i.e. $A^2 = 0$. Which of the following is true?

a. $A + \mathbf{1}$ is invertible and $(A + \mathbf{1})^{-1} = A - \mathbf{1}$
b. $A + \mathbf{1}$ is invertible and $(A + \mathbf{1})^{-1} = \mathbf{1} - \mathbf{2A}$
c. $A + \mathbf{1}$ is invertible and $(A + \mathbf{1})^{-1} = \mathbf{1} - A$
d. $A + \mathbf{1}$ is invertible and $(A + \mathbf{1})^{-1} = \mathbf{1} + A$
e. $A + \mathbf{1}$ is not invertible

3. Consider the vector space $M(2,2)$ of 2×2 matrices with the basis

$$E_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad E_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad E_3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad E_4 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Which of the following are the components of the matrix $A = \begin{pmatrix} 9 & 5 \\ 1 & 1 \end{pmatrix}$ relative to the E basis?

a. $(A)_E = \begin{pmatrix} 2 & 1 & 4 & 3 \end{pmatrix}^T$
b. $(A)_E = \begin{pmatrix} 5 & 4 & 3 & 2 \end{pmatrix}^T$
c. $(A)_E = \begin{pmatrix} 4 & 5 & 2 & 3 \end{pmatrix}^T$
d. $(A)_E = \begin{pmatrix} 1 & 2 & 3 & 4 \end{pmatrix}^T$
e. $(A)_E = \begin{pmatrix} 5 & -4 & 3 & -2 \end{pmatrix}^T$

4. Consider the vector space $C_2([0,1]$ of real valued function on the interval $[0,1]$ whose second derivatives exist and are continuous with the inner product $\langle f,g \rangle = \int_{-1}^{1} f(x)\,g(x)\,dx$. Consider the subspace $V = Span(x, x^2)$ spanned by the basis $v_1 = x$, $v_2 = x^2$. Which of the following is an orthonormal basis for V?

a. $u_1 = x \qquad u_2 = x^2$
b. $u_1 = \sqrt{\frac{5}{2}}\,x \qquad u_2 = \sqrt{\frac{7}{2}}\,x^2$
c. $u_1 = x \qquad u_2 = x^2 - \frac{3}{4}x$
d. $u_1 = \sqrt{\frac{3}{2}}\,x \qquad u_2 = \sqrt{\frac{5}{2}}\,x^2$
e. $u_1 = \sqrt{2}\,x \qquad u_2 = 4\sqrt{5}\,x^2 - 3\sqrt{5}\,x$

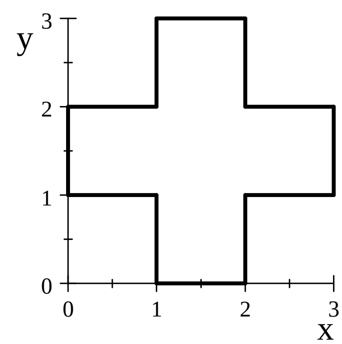
5. Consider the second derivative linear operator $L : P_6 \rightarrow P_6 : L(p) = \dfrac{d^2p}{dx^2}$ on the space of polynomials of degree less than 6. Find the image, $\mathrm{Im}(L)$.
HINT: Let $p = a + bx + cx^2 + dx^3 + ex^4 + fx^5$.

a. $\mathrm{Im}(L) = Span(1)$
b. $\mathrm{Im}(L) = Span(1, x)$
c. $\mathrm{Im}(L) = Span(1, x, x^2, x^3)$
d. $\mathrm{Im}(L) = Span(x^2, x^3, x^4, x^5)$
e. $\mathrm{Im}(L) = Span(x, x^2, x^3, x^4, x^5)$

6. Consider the second derivative linear operator $L : P_6 \rightarrow P_6 : L(p) = \dfrac{d^2p}{dx^2}$ on the space of polynomials of degree less than 6. Find the kernel, $Ker(L)$.
HINT: Let $p = a + bx + cx^2 + dx^3 + ex^4 + fx^5$.

a. $Ker(L) = Span(1)$
b. $Ker(L) = Span(1, x)$
c. $Ker(L) = Span(1, x, x^2)$
d. $Ker(L) = Span(x^2, x^3, x^4, x^5)$
e. $Ker(L) = Span(x, x^2, x^3, x^4, x^5)$

7. Compute the line integral $\oint \vec{F} \cdot d\vec{s}$ clockwise around the complete boundary of the plus sign, shown at the right, for the vector field $\vec{F} = (4x^3 + 2y, 4y^3 - 3x)$.



a. -25
b. -5
c. 0
d. 5
e. 25

8. Consider the vector space $M(2,2)$ of 2×2 matrices. Let $A = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$. Consider the linear function, $L : M(2,2) \to M(2,2) : L(X) = AX - XA$. Which of the following is not an eigenvalue and corresponding eigenmatrix (eigenvector) of L?

HINT: Let $X = \begin{pmatrix} x & y \\ z & w \end{pmatrix}$.

a. $\lambda = 2 \qquad X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

b. $\lambda = 1 \qquad X = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

c. $\lambda = 0 \qquad X = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

d. $\lambda = 0 \qquad X = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

e. $\lambda = -1 \qquad X = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

9. (16 points) Which of the following is an inner product on $\mathbb{R}^2$? If not, why not?

Put $\times$'s in the correct boxes. No part credit.

Let $\vec{x} = (x_1, x_2)$, $\vec{y} = (y_1, y_2)$.

		Inner Product?		Why not?			
				Not	Not	Not	Positive but Not
	$\langle \vec{x}, \vec{y} \rangle =$	Yes	No	Symmetric	Linear	Positive	Positive Definite
a.	$x_1y_1 + 2x_2y_2$						
b.	$x_1y_1 + 2x_1y_2 + 2x_2y_2$						
c.	$x_1^2y_1^2 + 2x_2^2y_2^2$						
d.	$x_1y_1 + x_1y_2 + x_2y_1 + 2x_2y_2$						
e.	$x_1y_1 - x_1y_2 - x_2y_1 + 2x_2y_2$						
f.	$x_1y_1 - x_1y_2 - x_2y_1 + x_2y_2$						
g.	$x_1y_1 + x_1y_2 + x_2y_1 + x_2y_2$						
h.	$x_1y_1 - x_2y_2$						

10. (20 points) Let $M(2,3)$ be the vector space of 2×3 matrices.

Consider the subspace $V = Span(A_1, A_2, A_3, A_4)$ where

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 3 & -2 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 2 & 2 & 0 \\ 4 & 6 & -4 \end{pmatrix}, \quad A_3 = \begin{pmatrix} -1 & 0 & 2 \\ -3 & 0 & 0 \end{pmatrix}, \quad A_4 = \begin{pmatrix} -1 & 2 & 6 \\ -5 & 6 & -4 \end{pmatrix}$$

Find a basis for V. What is the $\dim V$?

11. (28 points) Compute $\iint_H \vec{F} \cdot d\vec{S}$ over the hemisphere $z = \sqrt{25 - x^2 - y^2}$ oriented upward, for the vector field $\vec{F} = (x^3 - 4y^2 - 4z^2, -4x^2 + y^3 - 4z^2, -4x^2 - 4y^2 + z^3)$.

HINT: Use Gauss' Theorem by following these steps:

a. Write out Gauss' Theorem for the Volume, V, which is the solid hemisphere $0 \le z \le \sqrt{25 - x^2 - y^2}$. Split up the boundary, ∂V, into two pieces, the hemisphere, H, and the disk, D, at the bottom. State orientations. Solve for the integral you want.

b. Compute the volume integral using spherical coordinates.

(continued)

c. Compute the other surface integral over D by parametrizing the disk, computing the tangent vectors and normal vector, checking the orientation, evaluating the vector field on the disk and doing the integral.

d. Solve for the original integral.