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1	/60
2	/20
3	/20

MATH 311 Exam 2 Spring 2001

Section 200 P. Yasskin

1. (60 points) Let P_n be the vector space of polynomials of degree $\le n$. Consider the linear map $I : P_1 \to P_2$ given by

$$I(p)(x) = 2\int_1^x p(t)\,dt$$

Hint: For some parts it may be useful to write $p(t) = a + bt \in P_1$ and/or $q(x) = \alpha + \beta x + \gamma x^2 \in P_2$.

a. (3) Identify the domain of I, a basis for the domain, and the dimension of the domain.

b. (3) Identify the codomain of I, a basis for the codomain, and the dimension of the codomain.

c. (5) Identify the kernel of I, a basis for the kernel, and the dimension of the kernel.

d. (5) Identify the range of I, a basis for the range, and the dimension of the range.

e. (2) Is the function I one-to-one? Why?

f. (2) Is the function I onto? Why?

g. (2) Verify the dimensions in a, b, c and d agree with the Nullity-Rank Theorem.

h. (5) Find the matrix of I relative to the standard bases: (Call it $\underset{E\leftarrow e}{A}$.)

$$e_1 = 1, \quad e_2 = t \quad \text{for} \quad P_1 \qquad \text{and} \qquad E_1 = 1, \quad E_2 = x, \quad E_3 = x^2 \quad \text{for} \quad P_2$$

i. (6) Another basis for P_1 is $f_1 = 1 + 2t$, $f_2 = 1 + 3t$. Find the change of basis matrices between the e and f bases. (Call them $\underset{f\leftarrow e}{C}$ and $\underset{e\leftarrow f}{C}$.) Be sure to identify which is which!

j. (6) Consider the polynomial $q = 3 + 4t$. Find $[q]_e$ and $[q]_f$, the components of q relative to the e and f bases, respectively.

k. (3) A polynomial r has components $[r]_f = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ relative to the f basis. What is r?

l. (5) Find the matrix of I relative to the f basis for P_1 and the E basis for P_2. (Call it $\underset{E\leftarrow f}{B}$.)

m. (5) Find $\underset{E\leftarrow f}{B}$ by a second method.

n. (6) A polynomial r has components $[r]_f = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ relative to the f basis. Find $[I(r)]_E$, the components of $I(r)$ relative to the E basis. What is $I(r)$?

o. (2) Find $I(r)$ by a second method.

2. (20 points) Consider a linear map $L : \mathbf{R}^n \to \mathbf{R}^p$ whose matrix is $A = \begin{pmatrix} 1 & -2 & 0 & 3 \\ 2 & -4 & 1 & 2 \\ 0 & 0 & 1 & -4 \end{pmatrix}$.

- **a.** (2) What are n and p?
- **b.** (6) Identify the kernel of L, a basis for the kernel, and the dimension of the kernel.
- **c.** (6) Identify the range of L, a basis for the range, and the dimension of the range.
- **d.** (2) Is the function L one-to-one? Why?
- **e.** (2) Is the function L onto? Why?
- **f.** (2) Verify the dimensions in a, b and c agree with the Nullity-Rank Theorem.

3. (20 points) Consider the parabolic coordinate system

$$x = u^2 - v^2 \qquad y = 2uv$$

- **a.** (4) Describe the u-coordinate curve for which $v = 2$.
  (Give an xy-equation and describe the shape.)
- **b.** (4) Find $\vec{e}_u$, the vector tangent to the u-curve at the point $(u,v) = (3,2)$.
- **c.** (4) Describe the v-coordinate curve for which $u = 3$.
  (Give an xy-equation and describe the shape.)
- **d.** (4) Find $\vec{e}_v$, the vector tangent to the v-curve at the point $(u,v) = (3,2)$.
- **e.** (4) Compute $|\vec{e}_u|$, $|\vec{e}_v|$ and $\vec{e}_u \cdot \vec{e}_v$. Find the angle between $\vec{e}_u$ and $\vec{e}_v$.