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1	/10	4	/10
2	/10	5	/60
3	/10		

MATH 311 Exam 2 Fall 2001

Section 200 P. Yasskin

1. $\left(10 \text{ points}\right)$ Let P_1 be the vector space of polynomials of degree ≤ 1. Suppose $L : P_1 \to \mathbf{R}$ is a linear map which satisfies

$$L(2+3t) = 1, \qquad L(1+4t) = -2.$$

Compute $L(5-2t)$.

2. $\left(10 \text{ points}\right)$ Which of the following is not a subspace of $C^1[-1,1]$? Why?

$$P = \left\{f \in C^1[-1,1] \mid f(-1) = f(1)\right\} \qquad Q = \left\{f \in C^1[-1,1] \mid \frac{f(-1)+f(1)}{2} = f(0)\right\}$$

$$R = \left\{f \in C^1[-1,1] \mid \int_0^1 f(t)\,dt = 1\right\} \qquad S = \left\{f \in C^1[-1,1] \mid f'(0) = f(0)\right\}$$

3. (10 points) Duke Skywater is flying the Millennium Eagle through the Asteroid Belt. At the current time, his position is $\vec{r} = (4, -1, 2)$ and his velocity is $\vec{v} = (3, 2, -1)$. He measures that the electric field and its Jacobian are currently

$$\vec{E} = \begin{pmatrix} 12 \\ 2 \\ 9 \end{pmatrix} \quad \text{and} \quad J\vec{E} = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 4 & 3 \\ 1 & 2 & 9 \end{pmatrix}.$$

Use a linear (affine) approximation to estimate what the electric field will be 2 sec from now.

4. (10 points) Let $L : R^5 \to R^4$ be a linear map whose matrix is A. If A is row reduced, one obtains the matrix

$$\begin{pmatrix} 1 & 3 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

What is the dimension of the kernel of L? What is the dimension of the image of L?
Be sure to explain why.

5. $\big($60 points$\big)$ Let $M(2,2)$ be the vector space of 2×2 matrices. Let P_2 be the vector space of polynomials of degree ≤ 2. Consider the linear map $L : M(2,2) \to P_2$ given by

$$L(M) = \begin{pmatrix} 1 & x \end{pmatrix} M \begin{pmatrix} 1 \\ x \end{pmatrix}$$

Hint: For some parts it may be useful to write $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and/or $p(x) = \alpha + \beta x + \gamma x^2$.

a. (3) Identify the domain of L, a basis for the domain, and the dimension of the domain.

b. (3) Identify the codomain of L, a basis for the codomain, and the dimension of the codomain.

c. (6) Identify the kernel of L, a basis for the kernel, and the dimension of the kernel.

d. (6) Identify the image of L, a basis for the image, and the dimension of the image.

e. (2) Is the function L one-to-one? Why?

f. (2) Is the function L onto? Why?

g. (2) Verify the dimensions in a, b, c and d agree with the Nullity-Rank Theorem.

h. (6) Find the matrix of L relative to the standard bases: (Call it $\underset{E \leftarrow e}{A}$.)

$$e_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad e_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad e_3 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad e_4 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{for} \quad M(2,2)$$

$$\text{and} \quad E_1 = 1, \quad E_2 = x, \quad E_3 = x^2 \quad \text{for} \quad P_2$$

i. (6) Another basis for P_2 is $F_1 = 1 + x$, $F_2 = 1 + x^2$, $F_3 = x + x^2$. Find the change of basis matrices between the E and F bases. (Call them $\underset{F \leftarrow E}{C}$ and $\underset{E \leftarrow F}{C}$.) Be sure to identify which is which!

j. (6) Consider the polynomial $q = 2 + 4x$. Find $[q]_E$ and $[q]_F$, the components of q relative to the E and F bases, respectively. Check $[q]_F$.

k. (5) Find the matrix of L relative to the e basis for $M(2,2)$ and the F basis for P_2. (Call it $\underset{F \leftarrow e}{B}$.)

l. (5) Find $\underset{F \leftarrow e}{B}$ by a second method.

m. (6) Consider the matrix $N = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. Find $[N]_E$, the components of N relative to the E basis and $[L(N)]_F$, the components of $L(N)$ relative to the F basis. Use $[L(N)]_F$ to find $L(N)$?

n. (2) Recompute $L(N)$ using the definition of L.