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1	/35
2	/35
3	/30

MATH 311 Exam 3 Fall 2001

Section 200 P. Yasskin

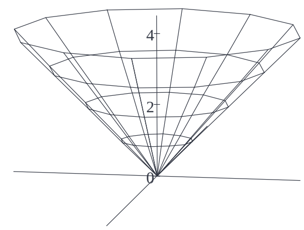
1. (35 points) Compute $\iint\limits_{\partial C} \vec{F} \cdot d\vec{S}$ over the complete surface of the box

$$0 \le x \le 2 \qquad 0 \le y \le 3 \qquad 0 \le z \le 4$$

where $\vec{F} = (x^2y^2z^3, xy^3z^3, xy^2z^4)$.

2. (35 points) Consider the cone C given by

$$z = \sqrt{x^2 + y^2} \quad \text{for } z \le 4$$



and the vector field $\vec{F} = (-yz, xz, -xy)$.

We want to compute $\iint\limits_C \vec{\nabla} \times \vec{F} \cdot d\vec{S}$ with normal pointing up and into the cone.

a. (5) Compute $\vec{\nabla} \times \vec{F}$.

b. (10) Parametrize the cone using cylindrical coordinates r and θ as the parameters and give the range of the parameters. Then explicitly compute $\iint\limits_C \vec{\nabla} \times \vec{F} \cdot d\vec{S}$.

RECALL: C is the cone $z = \sqrt{x^2 + y^2}$ for $z \le 4$ with normal pointing up and into the cone and $\vec{F} = (-yz, xz, -xy)$.

c. (10) Describe 2 other ways to compute $\iint_C \vec{\nabla} \times \vec{F} \cdot d\vec{S}$. Be sure to name or quote any Theorem you use and discuss the orientation of any curves or surfaces.

i.

ii.

d. (10) Recompute $\iint_C \vec{\nabla} \times \vec{F} \cdot d\vec{S}$ by **one** of these two methods.

3. $\big($30 points$\big)$ A hypersurface S in $\mathbf{R}^4$ with coordinates (w,x,y,z), may be parametrized by

$$(w,x,y,z) = \vec{R}(r,\theta,\varphi) = (r\cos\theta, r\sin\theta, r\cos\varphi, r\sin\varphi)$$

for $0 \le r \le 3, \quad 0 \le \theta \le 2\pi$ and $0 \le \varphi \le 2\pi$.

a. (15) Find the tangent vectors, the normal vector and length of the normal vector.

$\vec{R}_r =$

$\vec{R}_\theta =$

$\vec{R}_\varphi =$

$\vec{N} =$

$\left|\vec{N}\right| =$

b. (5) Find the hyperarea of the hypersurface.

$A =$

RECALL: S is the hypersurface parametrized by

$$(w,x,y,z) = \vec{R}(r,\theta,\varphi) = (r\cos\theta, r\sin\theta, r\cos\varphi, r\sin\varphi)$$

for $0 \le r \le 3$, $0 \le \theta \le 2\pi$ and $0 \le \varphi \le 2\pi$.

c. (5) Compute $P = \iiint\limits_S \sqrt{2w^2 + 2x^2}\, dS$ over the hypersurface.

d. (5) Compute $Q = \iiint\limits_S (w\,dy\,dx\,dz - 5z\,dw\,dx\,dy)$ over the hypersurface.