Name______________________ ID__________

1	/10	4	/20
2	/20	5-7	/30
3	/20		

MATH 311 Final Exam Fall 2001
Section 200 P. Yasskin

1. (10 points) Let P_2^0 be the subset of P_2 consisting of those polynomials of degree 2 or less whose constant term is zero. In particular

$$P_2^0 = \{p = ax + bx^2\}$$

a. (8) Show P_2^0 is a subspace of P_2.

b. (2) What is the dimension of P_2^0? Why?

2. (20 points) Again let P_2^0 be the subset of P_2 consisting of those polynomials of degree 2 or less whose constant term is zero. Consider the function $\langle *, * \rangle$ of two polynomials given by

$$\langle p, q \rangle = \int_0^1 \frac{4pq}{x^2}\, dx.$$

a. (5) Show the function $\langle *, * \rangle$ is an inner product on P_2^0.

b. (10) Apply the Gram Schmidt procedure to the basis $p_1 = x, \quad p_2 = x^2$ to produce an orthogonal basis q_1, q_2 and an orthonormal basis r_1, r_2.

$\Leftarrow \quad \Leftarrow \quad \Leftarrow$ USE THE BACK OF THE OPPOSITE PAGE.

$q_1 =$ $\qquad$ $q_2 =$ $\qquad$ $r_1 =$ $\qquad$ $r_2 =$

c. (5) Find the change of basis matrices $\underset{r \leftarrow p}{C}$ and $\underset{p \leftarrow r}{C}$.

3. (20 points) Again let P_2^0 be the subset of P_2 consisting of those polynomials of degree 2 or less whose constant term is zero. Consider the function $L : P_2^0 \to P_2$ given by

$$L(p) = p - \frac{dp}{dx}.$$

a. (4) Show the function L is linear.

b. (6) Find the kernel of L. Give a basis.

c. (6) Find the image of L. Give a basis.

d. (2) Is L onto? Why?

e. (2) Is L one-to-one? Why?

4. (20 points) Again let P_2^0 be the subset of P_2 consisting of those polynomials of degree 2 or less whose constant term is zero. Again consider the function $L : P_2^0 \to P_2$ given by

$$L(p) = p - \frac{dp}{dx}.$$

a. (10) Find the matrix of L relative to the bases

$$p_1 = x, \quad p_2 = x^2 \quad \text{for } P_2^0 \qquad \text{and} \qquad e_1 = 1, \quad e_2 = x, \quad e_3 = x^2 \quad \text{for } P_2.$$

Call it A.

b. (5) Find the matrix of L relative to the bases

$$r_1, r_2 \quad \text{for } P_2^0 \qquad \text{and} \qquad e_1 = 1, \quad e_2 = x, \quad e_3 = x^2 \quad \text{for } P_2$$

where r_1, r_2 is the orthonormal basis you found in problem 2. Call it B.

c. (5) Recompute B by another method.

5. $\left(30 \text{ points}\right)$ Do this problem, if you did the Volume of Desserts or Planet X Project.

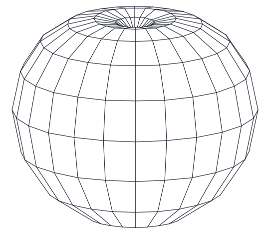
Find the z-component of the center of mass of the apple whose surface is given in spherical coordinates by

$$\rho = 1 - \cos\varphi$$

and whose density is 1.

HINT: The φ-integrals can be done using the substitution

$$u = 1 - \cos\varphi.$$



6. $\left(30 \text{ points}\right)$ Do this problem, if you did the Interpretation of Div and Curl Project.

Find the divergence of the vector field $\vec{F} = (xz^2, yz^2, 0)$ at the point $(x,y,z) = (0,0,c)$.

a. by using the derivative defintion:

b. by using the integral definition:

HINTS: For a sphere of radius ρ centered at (a,b,c), if you use standard spherical coordinates, the normal vector is

$$\vec{N} = \left(\rho^2 \sin^2\varphi \cos\theta, \rho^2 \sin^2\varphi \sin\theta, \rho^2 \cos\varphi \sin\varphi\right)$$

The φ-integral can be done using the substitution $u = \cos\varphi$.

You can ignore terms in the integral proportional to ρ^n with $n > 3$ since they drop out of the limit.

7. $\left(30 \text{ points}\right)$ Do this problem, if you did the Gauss' and Ampere's Laws Project.

Find the total charge in the cylinder $x^2 + y^2 \le a^2, \quad 0 \le z \le 1$ if the electric field is

$$\vec{E} = \frac{\hat{r}}{r} = \frac{\vec{r}}{r^2} = \left(\frac{x}{x^2+y^2}, \frac{y}{x^2+y^2}, 0\right)$$

where $\vec{r} = (x,y,0)$ and $r = \sqrt{x^2+y^2}$.

a. using the derivative form of Gauss' Law.

b. using the integral form of Gauss' Law.

c. What do these results tell you about the location of the electric charge? Why?