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MATH 311 Final Exam Fall 2002

Section 200 P. Yasskin

1-3	/30	6	/20
4	/20	7	/10
5	/20	8	/10

Multiple Choice: (10 points each) Work Out: (points indicated) Extra Credit: (10 points)

1. (10 points) If $L : P_2 \to \mathbb{R}$ is a linear function satisfying

$$L(1 + x + x^2) = 2 \qquad L(x + x^2) = -1 \qquad \text{and} \qquad L(x^2) = 3$$

find $L(2 + 3x + 5x^2)$.

a. 16
b. 9
c. 4
d. 0
e. -4

2. (10 points) Find the plane tangent to the hyperbolic paraboloid $x - yz = 0$ at the point $P = (6,3,2)$. Which of the following points does **not** lie on this plane?

a. $(-6,0,0)$
b. $(0,3,0)$
c. $(0,0,2)$
d. $(-1,1,1)$
e. $(1,-1,-1)$

3. (10 points) Duke Skywater is flying the Millenium Eagle through a polaron field. His galactic coordinates are $(2300,4200,1600)$ measured in lightseconds and his velocity is $\vec{v} = (.2,.3,.4)$ measured in lightseconds per second. He measures the strength of the polaron field is $p = 274$ milliwookies and its gradient is $\vec{\nabla} p = (3,2,2)$ milliwookies per lightsecond. Assuming a linear approximation for the polaron field and that his velocity is constant, how many seconds will Duke need to wait until the polaron field has grown to 286 milliwookies?

a. 2
b. 3
c. 4
d. 6
e. 12

4. (20 points) Consider the linear map $f : \mathbb{R}^5 \to \mathbb{R}^3$ given by $f(\vec{x}) = A\vec{x}$ where

$$A = \begin{pmatrix} 1 & 2 & -1 & 3 & 1 \\ 1 & 2 & 0 & 3 & -1 \\ -2 & -4 & 0 & -6 & 2 \end{pmatrix}.$$

When necessary, let $\vec{x} \in \mathbb{R}^5$ be $\vec{x} = \begin{pmatrix} r \\ s \\ t \\ u \\ v \end{pmatrix}$ and $\vec{z} \in \mathbb{R}^3$ be $\vec{z} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

a. (2) Identify the domain and give its dimension.

b. (2) Identify the codomain and give its dimension.

c. (2) Verify that f is linear.

d. (4) Find the kernel of f. Write it as a $Span$ and give a basis and its dimension.

e. (4) Find the image of f. Write it as a $Span$ and give a basis and its dimension.

f. (2) Verify your answers are consistent with the Nullity-Rank Theorem.

g. (2) Is f one-to-one? Why?

h. (2) Is f onto? Why?

5. $(20$ points$)$ Consider the vector space P_2 of polynomials of degree ≤ 2.
Consider the bases

$$e_1 = 1 \qquad e_2 = x \qquad e_3 = x^2$$

$$f_1 = 1 + x \qquad f_2 = x \qquad f_3 = -x + x^2$$

Consider the function $L : P_2 \to P_2$ given by

$$L(p) = 2p(0) + p(1)x$$

a. (4) Find the matrix of L relative to the e-basis (on both the domain and the codomain). Call it $\underset{e \leftarrow e}{A}$.

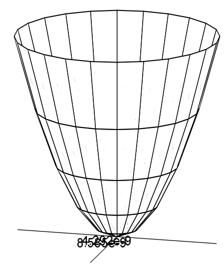
b. (8) Find the change of basis matrices between the e and f bases. (Call them $\underset{f \leftarrow e}{C}$ and $\underset{e \leftarrow f}{C}$.)
Be sure to identify which is which!

c. (4) Find the matrix of L relative to the f-basis. Call it $\underset{f \leftarrow f}{B}$.

d. (4) Find $\underset{f \leftarrow f}{B}$ by a second method.

6. $(20$ points$)$ **Stokes' Theorem** states that if S is a nice surface in $\mathbf{R}^3$ and ∂S is its boundary curve traversed counterclockwise as seen from the tip of the normal to S and $\vec{F}$ is a nice vector field on S then

$$\iint_S \vec{\nabla} \times \vec{F} \cdot d\vec{S} = \oint_{\partial S} \vec{F} \cdot d\vec{s}$$



Verify Stokes' Theorem if

$F = (y, -x, x^2 + y^2)$

and S is the paraboloid $z = x^2 + y^2$ for $z \le 4$

with **normal pointing up and in**.

Remember to check the orientations.

The paraboloid may be parametrized by:

$\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r^2)$

a. (10) Compute $\iint_S \vec{\nabla} \times \vec{F} \cdot d\vec{S}$ using the following steps:

$\vec{\nabla} \times \vec{F} =$

$\left(\vec{\nabla} \times \vec{F}\right)\left(\vec{R}(r,\theta)\right) =$

$\vec{R}_r =$

$\vec{R}_\theta =$

$\vec{N} =$

$\iint_S \vec{\nabla} \times \vec{F} \cdot d\vec{S} =$

b. (10) Recall $F = (y, -x, x^2 + y^2)$ and S is the paraboloid $z = x^2 + y^2$ for $z \le 4$ with **normal pointing up and in**. Compute $\oint_{\partial S} \vec{F} \cdot d\vec{s}$ using the following steps:

$\vec{r}(\theta) =$

$\vec{v}(\theta) =$

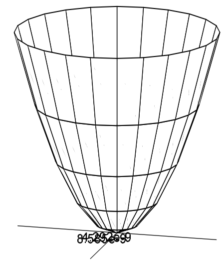
$\vec{F}(\vec{r}(\theta)) =$

$\oint_{\partial S} \vec{F} \cdot d\vec{s} =$

7. (10 points) The paraboloid at the right is the graph of the equation $z = x^2 + y^2$.
It may be parametrized as

$$\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r^2).$$

Find the area of the paraboloid for $z \le 4$.



8. (10 points) A paraboloid in $\mathbf{R}^4$ with coordinates (w,x,y,z), may be parametrized by

$$(w,x,y,z) = \vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r^2, r^2) \quad \text{for} \quad 0 \le r \le 3 \quad \text{and} \quad 0 \le \theta \le 2\pi.$$

Compute $I = \iint (xz\,dw\,dy - wy\,dx\,dz)$ over the surface.