Name_____ ID_____

MATH 311 Exam 1 Spring 2003

Section 200 P. Yasskin

1	/10	4	/15
2	/10	5	/25
3	/20	6	/20

If you use row or column operations, be sure to give your reasons.

1. $(10$ points$)$ Find a parametric equation for the line tangent to the curve $\vec{r}(t) = (t, t^2, t^3)$ at the point where $t = 2.$

2. $(10$ points$)$ Find the non-parametric equation for the plane tangent to the surface $x^3y^2 + xz^3 = 31$ at the point $(x, y, z) = (1, 2, 3).$

3. $(20$ points$)$ The following is a parametric surface in $\mathbb{R}^4$:

$$(w,x,y,z) = \vec{R}(u,v) = \left(u\sqrt{2}\cos v,\ u\sqrt{2}\sin v,\ v\cos u,\ v\sin u\right)$$

a. Find the two tangent vectors $\vec{e}_u$ and $\vec{e}_v$ at the point where $(u,v) = \left(\frac{\pi}{2}, \frac{\pi}{4}\right)$.

b. Find a parametric equation for the plane tangent to the surface $\vec{R}(u,v)$ at the point where $(u,v) = \left(\frac{\pi}{2}, \frac{\pi}{4}\right)$.

4. $(15$ points$)$ Let

$$M = \begin{pmatrix} 2 & 5 & 4 & -1 \\ 0 & 1 & -2 & 1 \\ 1 & 3 & 0 & -2 \\ 2 & 6 & 3 & x \end{pmatrix}$$

a. Compute $\det M$ (as a function of x).

b. For what value(s) of x does M^{-1} exist? Why?

5. $(25$ points$)$ Let $A = \begin{pmatrix} 3 & 2 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 2 \end{pmatrix}$.

a. Compute A^{-1}. Check it.

b. Solve the equations

$$\begin{aligned} 3x + 2y &= 2 \\ x - z &= 1 \\ y + 2z &= 3 \end{aligned}$$

6. $(20$ points$)$ (Multiple Choice: Circle one) If $C = AB$, then $(C^{\mathsf{T}})^{-1} =$

a. $A^{\mathsf{T}}B^{-1} + A^{-1}B^{\mathsf{T}}$

b. $B^{-1}A^{\mathsf{T}} + B^{\mathsf{T}}A^{-1}$

c. $(A^{-1})^{\mathsf{T}}(B^{-1})^{\mathsf{T}}$

d. $(B^{\mathsf{T}})^{-1}(A^{\mathsf{T}})^{-1}$

Now prove it. You may use any result proved in class or in the book or on homework.