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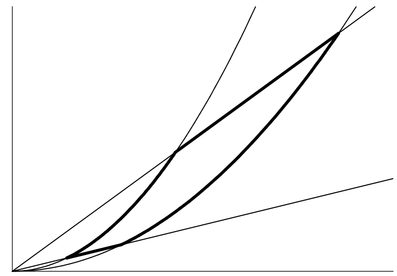
MATH 311 Exam 3 Spring 2003

Section 200 P. Yasskin

**1.** (15 points) Consider the "diamond shaped" region $D$ bounded by the lines $y = x$ and $y = 3x$ and the parabolas $y = x^2$ and $y = 2x^2$.

Compute $\iint_D \frac{y}{x}\,dA$ over the diamond.



Here are some steps to follow:

- Let $u = \frac{y}{x}$ and $v = \frac{y}{x^2}$. Solve for $x$ and $y$.

- Find the Jacobian factor.

- Express the integrand in terms of $u$ and $v$.

- Express the boundary curves in terms of $u$ and $v$.

- Compute $\iint_D \frac{y}{x}\,dA$.

**2.** (25 points) Consider the parametric curve $r(t) = \left(2t, t^2, \frac{1}{3}t^3\right)$ for $0 \le t \le 2$.

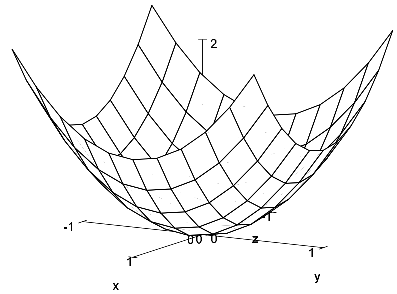
**a.** (15 pts) Compute $\int_{(0,0,0)}^{(4,4,8/3)} (xy + 3z)\, ds$ along this curve

**b.** (10 pts) Compute $\int_{(0,0,0)}^{(4,4,8/3)} \vec{F} \cdot d\vec{s}$ along this curve where $\vec{F} = (3z, 2y, x)$.

**3.** (30 points) Consider the parametric surface

$$\vec{R}(p,q) = (p, q, p^2 + q^2)$$

for $-1 \le p \le 1$ and $-1 \le q \le 1$.




**a.** (15 pts) Find the total mass $M = \iint \delta \, dS$ on this surface if the surface density is $\delta = \sqrt{4z+1}$.

**b.** (15 pts) Find the flux $\iint \vec{F} \cdot d\vec{S}$ of the vector field $\vec{F} = (3x, 3y, 3z)$ through this surface with normal pointing down.

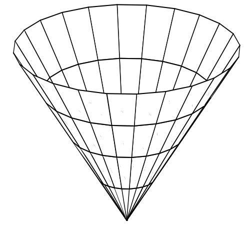
**4.** (30 points) Use 2 methods to compute

$$\iint_C \vec{F} \cdot d\vec{S} \quad \text{for} \quad \vec{F} = (5xz, 5yz, z^2)$$

over the conical surface $C$ given by

$$z = \sqrt{x^2 + y^2} \le 3$$

with normal pointing down and out.



**a.** (15 pts) METHOD 1: Compute $\iint_C \vec{F} \cdot d\vec{S}$ directly as a surface integral using the parametrization $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r)$.

HINT: Find $\vec{e}_r$, $\vec{e}_\theta$, $\vec{N}$ and $\vec{F}$ on the cone.

Recall: $\vec{F} = (5xz, 5yz, z^2)$ and $C$ is the conical surface $z = \sqrt{x^2 + y^2} \le 3$ with normal pointing down and out.

**b.** (15 pts) METHOD 2: Compute $\iint_C \vec{F} \cdot d\vec{S}$ by applying Gauss' Theorem

$\iiint_V \vec{\nabla} \cdot \vec{F}\, dV = \iint_{\partial V} \vec{F} \cdot d\vec{S}$ to the solid cone $V$ whose boundary is $\partial V = C + D$

where $C$ is the conical surface and $D$ is the disk at the top of the cone.