Consider the 2-dimensional manifold with Lorentz signature metric

$$ds^2 = -dt^2 + (1+t)^n dx^2$$

Use Latin indices $i,j,\ldots$ to denote $0 = t$ and $1 = x$.

1. Write out the components of the metric, its inverse and its determinant.

2. Compute the connection coefficients:

$$\Gamma^i{}_{jk} = \frac{1}{2} g^{il}(g_{lj,k} + g_{lk,j} - g_{jk,l})$$

3. Compute the Riemann curvature:

$$R^{i}_{\ jkl} = \partial_k \Gamma^{i}_{\ jl} - \partial_l \Gamma^{i}_{\ jk} + \Gamma^{i}_{\ mk} \Gamma^{m}_{\ jl} - \Gamma^{i}_{\ ml} \Gamma^{m}_{\ jk}$$

4. Compute the Ricci curvature:

$$R_{ij} = R^{k}_{ikj}$$

5. Compute the Scalar curvature:

$$R = R^{i}_{i}$$

6. For which values of n is the scalar curvature zero, positive and negative?

7. Find the area of the region $0 \leq t \leq T$ and $0 \leq x \leq X$.