$$R^d_{abc} = \partial_b \Gamma^d_{ac} - \partial_c \Gamma^d_{ab} + \Gamma^d_{eb} \Gamma^e_{ac} - \Gamma^d_{ec} \Gamma^e_{ab}$$
$$R^d_{bca} = \partial_c \Gamma^d_{ba} - \partial_a \Gamma^d_{bc} + \Gamma^d_{ec} \Gamma^e_{ba} - \Gamma^d_{ea} \Gamma^e_{bc}$$
$$R^d_{cab} = \partial_a \Gamma^d_{cb} - \partial_b \Gamma^d_{ca} + \Gamma^d_{ea} \Gamma^e_{cb} - \Gamma^d_{eb} \Gamma^e_{ca}$$
$$R^d_{abc} + R^d_{bca} + R^d_{cab} = 0 \qquad \text{1}^{\text{st}} \text{ Bianchi Identity}$$

$$R^m_{nab} = \partial_a \Gamma^m_{nb} - \partial_b \Gamma^m_{na} + \Gamma^m_{ea} \Gamma^e_{nb} - \Gamma^m_{eb} \Gamma^e_{na}$$

$$R^m_{nab;c} = \partial_c R^m_{nab} + \Gamma^m_{kc} R^k_{nab} - \Gamma^k_{nc} R^m_{kab} - \Gamma^k_{ac} R^m_{nkb} - \Gamma^k_{bc} R^m_{nak}$$

$$R^m_{nab;c} =$$
$$\partial_c \partial_a \Gamma^m_{nb} - \partial_c \partial_b \Gamma^m_{na} + \partial_c \Gamma^m_{ea} \Gamma^e_{nb} + \Gamma^m_{ea} \partial_c \Gamma^e_{nb} - \partial_c \Gamma^m_{eb} \Gamma^e_{na} - \Gamma^m_{eb} \partial_c \Gamma^e_{na}$$
$$+ \Gamma^m_{kc} (\partial_a \Gamma^k_{nb} - \partial_b \Gamma^k_{na} + \Gamma^k_{ea} \Gamma^e_{nb} - \Gamma^k_{eb} \Gamma^e_{na})$$
$$- \Gamma^k_{nc} (\partial_a \Gamma^m_{kb} - \partial_b \Gamma^m_{ka} + \Gamma^m_{ea} \Gamma^e_{kb} - \Gamma^m_{eb} \Gamma^e_{ka})$$
$$- \Gamma^k_{ac} (\partial_k \Gamma^m_{nb} - \partial_b \Gamma^m_{nk} + \Gamma^m_{ek} \Gamma^e_{nb} - \Gamma^m_{eb} \Gamma^e_{nk})$$
$$- \Gamma^k_{bc} (\partial_a \Gamma^m_{nk} - \partial_k \Gamma^m_{na} + \Gamma^m_{ea} \Gamma^e_{nk} - \Gamma^m_{ek} \Gamma^e_{na})$$

$$R^m_{nbc;a} =$$
$$\partial_a \partial_b \Gamma^m_{nc} - \partial_a \partial_c \Gamma^m_{nb} + \partial_a \Gamma^m_{eb} \Gamma^e_{nc} + \Gamma^m_{eb} \partial_a \Gamma^e_{nc} - \partial_a \Gamma^m_{ec} \Gamma^e_{nb} - \Gamma^m_{ec} \partial_a \Gamma^e_{nb}$$
$$+ \Gamma^m_{ka} (\partial_b \Gamma^k_{nc} - \partial_c \Gamma^k_{nb} + \Gamma^k_{eb} \Gamma^e_{nc} - \Gamma^k_{ec} \Gamma^e_{nb})$$
$$- \Gamma^k_{na} (\partial_b \Gamma^m_{kc} - \partial_c \Gamma^m_{kb} + \Gamma^m_{eb} \Gamma^e_{kc} - \Gamma^m_{ec} \Gamma^e_{kb})$$
$$- \Gamma^k_{ba} (\partial_k \Gamma^m_{nc} - \partial_c \Gamma^m_{nk} + \Gamma^m_{ek} \Gamma^e_{nc} - \Gamma^m_{ec} \Gamma^e_{nk})$$
$$- \Gamma^k_{ca} (\partial_b \Gamma^m_{nk} - \partial_k \Gamma^m_{nb} + \Gamma^m_{eb} \Gamma^e_{nk} - \Gamma^m_{ek} \Gamma^e_{nb})$$

$$R^m_{nca;b} =$$
$$\partial_b \partial_c \Gamma^m_{na} - \partial_b \partial_a \Gamma^m_{nc} + \partial_b \Gamma^m_{ec} \Gamma^e_{na} + \Gamma^m_{ec} \partial_b \Gamma^e_{na} - \partial_b \Gamma^m_{ea} \Gamma^e_{nc} - \Gamma^m_{ea} \partial_b \Gamma^e_{nc}$$
$$+ \Gamma^m_{kb} (\partial_c \Gamma^k_{na} - \partial_a \Gamma^k_{nc} + \Gamma^k_{ec} \Gamma^e_{na} - \Gamma^k_{ea} \Gamma^e_{nc})$$
$$- \Gamma^k_{nb} (\partial_c \Gamma^m_{ka} - \partial_a \Gamma^m_{kc} + \Gamma^m_{ec} \Gamma^e_{ka} - \Gamma^m_{ea} \Gamma^e_{kc})$$
$$- \Gamma^k_{cb} (\partial_k \Gamma^m_{na} - \partial_a \Gamma^m_{nk} + \Gamma^m_{ek} \Gamma^e_{na} - \Gamma^m_{ea} \Gamma^e_{nk})$$
$$- \Gamma^k_{ab} (\partial_c \Gamma^m_{nk} - \partial_k \Gamma^m_{nc} + \Gamma^m_{ec} \Gamma^e_{nk} - \Gamma^m_{ek} \Gamma^e_{nc})$$

$$R^m_{nab;c} + R^m_{nbc;a} + R^m_{nca;b} =$$

$$\partial_c\partial_a\Gamma^m_{nb} - \partial_c\partial_b\Gamma^m_{na} + \partial_a\partial_b\Gamma^m_{nc} - \partial_a\partial_c\Gamma^m_{nb} + \partial_b\partial_c\Gamma^m_{na} - \partial_b\partial_a\Gamma^m_{nc}$$

$$+[\Gamma^m_{kc}\partial_a\Gamma^k_{nb}]_1 - [\Gamma^k_{nc}\partial_a\Gamma^m_{kb}]_2 - [\Gamma^k_{bc}\partial_a\Gamma^m_{nk}]_3$$

$$+[\partial_a\Gamma^m_{eb}\Gamma^e_{nc}]_2 + [\Gamma^m_{eb}\partial_a\Gamma^e_{nc}]_4 - [\partial_a\Gamma^m_{ec}\Gamma^e_{nb}]_5 - [\Gamma^m_{ec}\partial_a\Gamma^e_{nb}]_1$$

$$-[\Gamma^m_{kb}\partial_a\Gamma^k_{nc}]_4 + [\Gamma^k_{nb}\partial_a\Gamma^m_{kc}]_5 + [\Gamma^k_{cb}\partial_a\Gamma^m_{nk}]_3$$

$$+(a \rightarrow b) + (a \rightarrow c)$$

$$+\Gamma^m_{kc}(\Gamma^k_{ea}\Gamma^e_{nb} - \Gamma^k_{eb}\Gamma^e_{na})$$

$$-\Gamma^k_{nc}(\Gamma^m_{ea}\Gamma^e_{kb} - \Gamma^m_{eb}\Gamma^e_{ka})$$

$$-\Gamma^k_{ac}(\Gamma^m_{ek}\Gamma^e_{nb} - \Gamma^m_{eb}\Gamma^e_{nk})$$

$$-\Gamma^k_{bc}(\Gamma^m_{ea}\Gamma^e_{nk} - \Gamma^m_{ek}\Gamma^e_{na})$$

$$+\Gamma^m_{ka}(\Gamma^k_{eb}\Gamma^e_{nc} - \Gamma^k_{ec}\Gamma^e_{nb})$$

$$-\Gamma^k_{na}(\Gamma^m_{eb}\Gamma^e_{kc} - \Gamma^m_{ec}\Gamma^e_{kb})$$

$$-\Gamma^k_{ba}(\Gamma^m_{ek}\Gamma^e_{nc} - \Gamma^m_{ec}\Gamma^e_{nk})$$

$$-\Gamma^k_{ca}(\Gamma^m_{eb}\Gamma^e_{nk} - \Gamma^m_{ek}\Gamma^e_{nb})$$

$$+\Gamma^m_{kb}(\Gamma^k_{ec}\Gamma^e_{na} - \Gamma^k_{ea}\Gamma^e_{nc})$$

$$-\Gamma^k_{nb}(\Gamma^m_{ec}\Gamma^e_{ka} - \Gamma^m_{ea}\Gamma^e_{kc})$$

$$-\Gamma^k_{cb}(\Gamma^m_{ek}\Gamma^e_{na} - \Gamma^m_{ea}\Gamma^e_{nk})$$

$$-\Gamma^k_{ab}(\Gamma^m_{ec}\Gamma^e_{nk} - \Gamma^m_{ek}\Gamma^e_{nc})$$

$$=$$

$$+[\Gamma^k_{ea}\Gamma^e_{nb}\Gamma^m_{kc}]_1 - [\Gamma^e_{na}\Gamma^k_{eb}\Gamma^m_{kc}]_2$$

$$-[\Gamma^m_{ea}\Gamma^e_{kb}\Gamma^k_{nc}]_3 + [\Gamma^e_{ka}\Gamma^m_{eb}\Gamma^k_{nc}]_4$$

$$+[\Gamma^m_{ka}\Gamma^k_{eb}\Gamma^e_{nc}]_3 - [\Gamma^m_{ka}\Gamma^e_{nb}\Gamma^k_{ec}]_5$$

$$-[\Gamma^k_{na}\Gamma^m_{eb}\Gamma^e_{kc}]_6 + [\Gamma^k_{na}\Gamma^e_{kb}\Gamma^m_{ec}]_2$$

$$+[\Gamma^e_{na}\Gamma^m_{kb}\Gamma^k_{ec}]_6 - [\Gamma^k_{ea}\Gamma^m_{kb}\Gamma^e_{nc}]_4$$

$$-[\Gamma^e_{ka}\Gamma^k_{nb}\Gamma^m_{ec}]_1 + [\Gamma^m_{ea}\Gamma^k_{nb}\Gamma^e_{kc}]_5$$

$$-[\Gamma^k_{ac}\Gamma^m_{ek}\Gamma^e_{nb}]_7 + [\Gamma^k_{ac}\Gamma^m_{eb}\Gamma^e_{nk}]_8$$

$$-[\Gamma^k_{ca}\Gamma^m_{eb}\Gamma^e_{nk}]_8 + [\Gamma^k_{ca}\Gamma^m_{ek}\Gamma^e_{nb}]_7$$

$$-[\Gamma^k_{bc}\Gamma^m_{ea}\Gamma^e_{nk}]_9 + [\Gamma^k_{bc}\Gamma^m_{ek}\Gamma^e_{na}]_{10}$$

$$-[\Gamma^k_{cb}\Gamma^m_{ek}\Gamma^e_{na}]_{10} + [\Gamma^k_{cb}\Gamma^m_{ea}\Gamma^e_{nk}]_9$$

$$-[\Gamma^k_{ba}\Gamma^m_{eb}\Gamma^e_{nc}]_{11} + [\Gamma^k_{ba}\Gamma^m_{ec}\Gamma^e_{nk}]_{12}$$

$$-[\Gamma^k_{ab}\Gamma^m_{ec}\Gamma^e_{nk}]_{12} + [\Gamma^k_{ab}\Gamma^m_{ek}\Gamma^e_{nc}]_{11}$$

$$R^m_{nab;c} + R^m_{nbc;a} + R^m_{nca;b} = 0 \qquad \text{2}^{\text{nd}}\text{ Bianchi Identity}$$

$$R^a_{nab;c} + R^a_{nbc;a} + R^a_{nca;b} = 0$$

$$R_{nb;c} + R^a_{nbc;a} - R_{nc;b} = 0$$

$$g^{nb}R_{nb;c} + g^{nb}R^a_{nbc;a} - g^{nb}R_{nc;b} = 0$$

$$R_{;c} - R^a_{c;a} - R^b_{c;b} = 0$$

$$R^a_{c;a} - \frac{1}{2}\delta^a_c R_{;a}$$

$$E^a_{b;a} = 0 \qquad \text{Contracted Bianchi Identity}$$