Sample problems for Test 1

Any problem may be altered or replaced by a different one!

Problem 1 (15 pts.) Find a quadratic polynomial $p(x)$ such that $p(1) = 1$, $p(2) = 3$, and $p(3) = 7$.

Problem 2 (25 pts.) Let $A = \begin{pmatrix} 1 & -2 & 4 & 1 \\ 2 & 3 & 2 & 0 \\ 2 & 0 & -1 & 1 \\ 2 & 0 & 0 & 1 \end{pmatrix}$.

(i) Evaluate the determinant of the matrix A.
(ii) Find the inverse matrix A^{-1}.

Problem 3 (20 pts.) Determine which of the following subsets of $\mathbb{R}^3$ are subspaces. Briefly explain.

(i) The set S_1 of vectors $(x, y, z) \in \mathbb{R}^3$ such that $xyz = 0$.
(ii) The set S_2 of vectors $(x, y, z) \in \mathbb{R}^3$ such that $x + y + z = 0$.
(iii) The set S_3 of vectors $(x, y, z) \in \mathbb{R}^3$ such that $y^2 + z^2 = 0$.
(iv) The set S_4 of vectors $(x, y, z) \in \mathbb{R}^3$ such that $y^2 - z^2 = 0$.

Problem 4 (30 pts.) Let $B = \begin{pmatrix} 0 & -1 & 4 & 1 \\ 1 & 1 & 2 & -1 \\ -3 & 0 & -1 & 0 \\ 2 & -1 & 0 & 1 \end{pmatrix}$.

(i) Find the rank and the nullity of the matrix B.
(ii) Find a basis for the row space of B, then extend this basis to a basis for $\mathbb{R}^4$.
(iii) Find a basis for the nullspace of B.

Bonus Problem 5 (15 pts.) Show that the functions $f_1(x) = x$, $f_2(x) = xe^x$, and $f_3(x) = e^{-x}$ are linearly independent in the vector space $C^\infty(\mathbb{R})$.

Bonus Problem 6 (15 pts.) Let V be a finite-dimensional vector space and V_0 be a proper subspace of V (where proper means that $V_0 \neq V$). Prove that $\dim V_0 < \dim V$.