

Sample problems for Test 1

Any problem may be altered or replaced by a different one!

Problem 1 (25 pts.) Let Π be the plane in \mathbb{R}^3 passing through the points $(2, 0, 0)$, $(1, 1, 0)$, and $(-3, 0, 2)$. Let ℓ be the line in \mathbb{R}^3 passing through the point $(1, 1, 1)$ in the direction $(2, 2, 2)$.

- (i) Find a parametric representation for the line ℓ .
- (ii) Find a parametric representation for the plane Π .
- (iii) Find an equation for the plane Π .
- (iv) Find the point where the line ℓ intersects the plane Π .
- (v) Find the angle between the line ℓ and the plane Π .
- (vi) Find the distance from the origin to the plane Π .

Problem 2 (15 pts.) Let $f(x) = a \sin x + b \cos x + c$. Find a , b , and c so that $f(0) = 1$, $f'(0) = 2$, and $f''(0) = 3$.

Problem 3 (20 pts.) Let $A = \begin{pmatrix} 3 & 5 \\ -2 & 1 \end{pmatrix}$. Compute the matrices A^2 , A^3 , and $p(A)$, where $p(x) = 2x^2 - 3x + 1$.

Problem 4 (20 pts.) Let $A = \begin{pmatrix} 5 & -2 & 4 \\ 4 & -3 & 2 \\ -3 & 4 & -1 \end{pmatrix}$. Find the inverse matrix A^{-1} .

Problem 5 (20 pts.) Evaluate the following determinants:

$$\begin{vmatrix} 5 & -2 & 4 \\ 4 & -3 & 2 \\ -3 & 4 & -1 \end{vmatrix}, \quad \begin{vmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{vmatrix}, \quad \begin{vmatrix} -1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 7 \end{vmatrix}.$$

Bonus Problem 6 (25 pts.) Find the volume of the parallelepiped bounded by the following three pairs of parallel planes in \mathbb{R}^3 :

- (1) $x + y = 2$ and $x + y = 4$,
- (2) $y + z = 3$ and $y + z = -3$,
- (3) $z = 0$ and $z = 5$.