Math 311-503 Spring 2007

Sample problems for Test 2

Any problem may be altered or replaced by a different one!

Problem 1 (20 pts.) Let $\mathcal{P}_2$ be the vector space of all polynomials (with real coefficients) of degree at most 2. Determine which of the following subsets of $\mathcal{P}_2$ are vector subspaces. Briefly explain.

(i) The set S_1 of polynomials $p(x) \in \mathcal{P}_2$ such that $p(0) = 0$.
(ii) The set S_2 of polynomials $p(x) \in \mathcal{P}_2$ such that $p(0) = 0$ and $p(1) = 0$.
(iii) The set S_3 of polynomials $p(x) \in \mathcal{P}_2$ such that $p(0) = 0$ or $p(1) = 0$.
(iv) The set S_4 of polynomials $p(x) \in \mathcal{P}_2$ such that $(p(0))^2 + 2(p(1))^2 + (p(2))^2 = 0$.

Problem 2 (20 pts.) Let L be the linear operator on $\mathbb{R}^2$ given by

$$L\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}.$$

Find the matrix of the operator L relative to the basis $\mathbf{v}_1 = (1, 1)$, $\mathbf{v}_2 = (1, -1)$.

Problem 3 (30 pts.) Consider a linear operator $f : \mathbb{R}^3 \to \mathbb{R}^3$, $f(\mathbf{x}) = A\mathbf{x}$, where

$$A = \begin{pmatrix} 5 & 3 & 5 \\ 2 & 1 & 2 \\ 1 & 0 & 1 \end{pmatrix}.$$

(i) Find a basis for the image of f.
(ii) Find a basis for the null-space of f.

Problem 4 (30 pts.) Let $B = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$.

(i) Find all eigenvalues of the matrix B.
(ii) For each eigenvalue of B, find an associated eigenvector.
(iii) Is there a basis for $\mathbb{R}^3$ consisting of eigenvectors of B?

Bonus Problem 5 (25 pts.) Let $f_1, f_2, f_3, \dots$ be the Fibonacci numbers defined by $f_1 = f_2 = 1$, $f_n = f_{n-1} + f_{n-2}$ for $n \geq 3$. Find $\lim\limits_{n\to\infty} \dfrac{f_{n+1}}{f_n}$.