Sample problems for Test 1

Any problem may be altered or replaced by a different one!

Problem 1 (25 pts.)   Let Π be the plane in $\mathbb{R}^3$ passing through the points $(2, 0, 0)$, $(1, 1, 0)$, and $(-3, 0, 2)$. Let ℓ be the line in $\mathbb{R}^3$ passing through the point $(1, 1, 1)$ in the direction $(2, 2, 2)$.

(i) Find a parametric representation for the line ℓ.
(ii) Find a parametric representation for the plane Π.
(iii) Find an equation for the plane Π.
(iv) Find the point where the line ℓ intersects the plane Π.
(v) Find the angle between the line ℓ and the plane Π.
(vi) Find the distance from the origin to the plane Π.

Problem 2 (15 pts.)   Let $f(x) = a \cos 2x + b \cos x + c$. Find a, b, and c so that $f(0) = 0$, $f''(0) = 2$, and $f''''(0) = 10$.

Problem 3 (20 pts.)   Let $A = \begin{pmatrix} 0 & -2 & 4 & 1 \\ 2 & 3 & 2 & 0 \\ 1 & 0 & -1 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix}$. Find the inverse matrix A^{-1}.

Problem 4 (20 pts.)   Evaluate the following determinants:

$$\text{(i)} \quad \begin{vmatrix} 0 & -2 & 4 & 1 \\ 2 & 3 & 2 & 0 \\ 1 & 0 & -1 & 1 \\ 1 & 0 & 0 & 1 \end{vmatrix}, \qquad \text{(ii)} \quad \begin{vmatrix} 2 & -2 & 0 & 3 \\ -5 & 3 & 2 & 1 \\ 1 & -1 & 0 & -3 \\ 2 & 0 & 0 & -1 \end{vmatrix}.$$

Bonus Problem 5 (15 pts.)   Find the volume of the tetrahedron with vertices at the points $\mathbf{a} = (1, 0, 0)$, $\mathbf{b} = (0, 1, 0)$, $\mathbf{c} = (0, 0, 1)$, and $\mathbf{d} = (2, 3, 5)$.