

### Sample problems for Test 3

Any problem may be altered or replaced by a different one!

**Problem 1** Find  $\text{curl}(\text{curl}(\mathbf{F}))$ , where  $\mathbf{F}(x, y, z) = (x^2 + y^2)\mathbf{e}_1 + ze^{x+y}\mathbf{e}_2 + (x + \sin y)\mathbf{e}_3$ .

**Problem 2** Evaluate a double integral

$$\iint_P (2x + 3y - \cos(\pi x + 2\pi y)) \, dx \, dy$$

over a parallelogram  $P$  with vertices  $(-1, -1)$ ,  $(1, 0)$ ,  $(2, 2)$ , and  $(0, 1)$ .

**Problem 3** Find the area of a cardioid which boundary is given by  $r = 1 - \cos \phi$  in polar coordinates.

**Problem 4** Consider a vector field  $\mathbf{F}(x, y, z) = (yz + 2 \cos 2x, xz - e^z, xy - ye^z)$ .

- (i) Verify that the field  $\mathbf{F}$  is conservative.
- (ii) Find a function  $f$  such that  $\mathbf{F} = \nabla f$ .

**Problem 5** Let  $C$  be a solid cylinder bounded by planes  $z = 0$ ,  $z = 2$  and a cylindrical surface  $x^2 + y^2 = 1$ . Orient the boundary  $\partial C$  with outward normals and evaluate a surface integral

$$\iint_{\partial C} (x^2\mathbf{e}_1 + y^2\mathbf{e}_2 + z^2\mathbf{e}_3) \cdot d\mathbf{S}.$$

**Problem 6** Let  $D$  be a region in  $\mathbb{R}^3$  bounded by a paraboloid  $z = x^2 + y^2$  and a plane  $z = 9$ . Let  $S$  denote the part of the paraboloid that bounds  $D$ , oriented by outward normals. Evaluate a surface integral

$$\iint_S \text{curl}(\mathbf{F}) \cdot d\mathbf{S},$$

where  $\mathbf{F}(x, y, z) = (e^{x^2+z^2}, xy + xz + yz, e^{xyz})$ .