Sample problems for Test 3

(to be worked out during the review)

Problem 1 Find curl(curl($\mathbf{F}$)), where $\mathbf{F}(x, y, z) = (x^2 + y^2)\mathbf{e}_1 + ze^{x+y}\mathbf{e}_2 + (x + \sin y)\mathbf{e}_3$.

Problem 2 Evaluate a double integral

$$\iint_P \big(2x + 3y - \cos(\pi x + 2\pi y)\big)\, dx\, dy$$

over a parallelogram P with vertices $(-1, -1)$, $(1, 0)$, $(2, 2)$, and $(0, 1)$.

Problem 3 Find the volume of a tetrahedron (i.e., triangular pyramid) with vertices at points $(0, 2, 1)$, $(1, 0, 0)$, $(2, 1, 2)$, and $(3, 1, 1)$.

Problem 4 Consider a vector field $\mathbf{F}(x, y, z) = (yz + 2\cos 2x, xz - e^z, xy - ye^z)$.

(i) Verify that the field $\mathbf{F}$ is conservative.
(ii) Find a function f such that $\mathbf{F} = \nabla f$.

Problem 5 Let C be a solid cylinder bounded by planes $z = 0$, $z = 2$ and a cylindrical surface $x^2 + y^2 = 1$. Orient the boundary ∂C with outward normals and evaluate a surface integral

$$\iint_{\partial C} (x^2\mathbf{e}_1 + y^2\mathbf{e}_2 + z^2\mathbf{e}_3) \cdot d\mathbf{S}.$$

Problem 6 Let D be a region in $\mathbb{R}^3$ bounded by a paraboloid $z = x^2 + y^2$ and a plane $z = 9$. Let S denote the part of the paraboloid that bounds D, oriented by outward normals. Evaluate a surface integral

$$\iint_S \operatorname{curl}(\mathbf{F}) \cdot d\mathbf{S},$$

where $\mathbf{F}(x, y, z) = (e^{x^2+z^2}, xy + xz + yz, e^{xyz})$.