

Homework assignment #10

Problem 1. Find the distance from the point $(1, 2)$ to the line $4x - 3y = 0$.

Problem 2. Let V be the subspace of \mathbb{R}^3 spanned by $(1, -1, 1)$. Find a basis for the orthogonal complement V^\perp .

Problem 3. Find the orthogonal complement of the subspace of \mathbb{R}^3 spanned by vectors $(1, 2, 1)$ and $(1, -1, 2)$.

Problem 4. Let V be the subspace of \mathbb{R}^4 spanned by vectors $(1, 0, -2, 1)$ and $(0, 1, 3, -2)$. Find a basis for the orthogonal complement V^\perp .

Problem 5. If A is an $m \times n$ matrix of rank r , what are the dimensions of the nullspaces $N(A)$ and $N(A^T)$? Explain.

Problem 6. For each of the following systems, find the least squares solution:

$$\begin{array}{ccc}
 \text{(i)} \begin{cases} x_1 + 2x_2 = 3 \\ 2x_1 + 4x_2 = 2 \\ -x_1 - 2x_2 = 1 \end{cases} &
 \text{(ii)} \begin{cases} -x_1 + x_2 = 10 \\ 2x_1 + x_2 = 5 \\ x_1 - 2x_2 = 20 \end{cases} &
 \text{(iii)} \begin{cases} x_1 + x_2 + x_3 = 4 \\ -x_1 + x_2 + x_3 = 0 \\ -x_2 + x_3 = 1 \\ x_1 + x_3 = 2 \end{cases}
 \end{array}$$

Problem 7. Consider the following data:

$$\begin{array}{c|c|c|c|c}
 x & -1 & 0 & 1 & 2 \\
 \hline
 f(x) & 0 & 1 & 3 & 9
 \end{array}$$

(i) Find a linear polynomial which is the best least squares fit to the data. Plot the graph along with the data on a coordinate system.

(ii) Find a quadratic polynomial which is the best least squares fit to the data. Plot the graph along with the data on a coordinate system.