MATH 323–502 Fall 2022

Homework assignment #6

Problem 1. Given $\mathbf{v}_1 = (1, 1, 1)$ and $\mathbf{v}_2 = (3, -1, 4)$, find a third vector $\mathbf{v}_3$ that will extend the set $\{\mathbf{v}_1, \mathbf{v}_2\}$ to a basis for $\mathbb{R}^3$.

Problem 2. The following vectors span the vector space $\mathbb{R}^3$:

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 2 \\ 5 \\ 4 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \quad \mathbf{v}_4 = \begin{pmatrix} 2 \\ 7 \\ 4 \end{pmatrix}, \quad \mathbf{v}_5 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}.$$

Pare down the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5\}$ to a basis for $\mathbb{R}^3$.

Problem 3. Find the dimension of the subspace of $\mathbb{R}^3$ spanned by the given vectors:
(i) $(1, -2, 2)$, $(2, -2, 4)$ and $(-3, 3, 6)$;
(ii) $(1, 1, 1)$, $(1, 2, 3)$ and $(2, 3, 1)$;
(iii) $(1, -1, 2)$, $(-2, 2, 4)$, $(3, -2, 5)$ and $(2, -1, 3)$.

Problem 4. Find the dimension of the subspace of $\mathcal{P}$ spanned by the given polynomials:
(i) x, $x - 1$, $x^2 + 1$ and $x^2 - 1$;
(ii) x^2, $x^2 - x - 1$ and $x + 1$.

Problem 5 (2 pts). Find a basis for the row space, a basis for the column space, and a basis for the nullspace of the following matrix:

$$\begin{pmatrix} 1 & 3 & 2 \\ 2 & 1 & 4 \\ 4 & 7 & 8 \end{pmatrix}.$$

Problem 6. Prove that a linear system $A\mathbf{x} = \mathbf{b}$ is consistent if and only if the rank of the augmented matrix $(A \,|\, \mathbf{b})$ equals the rank of the coefficient matrix A.

[Hint: compare the column spaces.]