Homework assignment #7

Problem 1. Vectors $\mathbf{v}_1 = (1, 1)$ and $\mathbf{v}_2 = (-1, 1)$ form a basis for the vector space $\mathbb{R}^2$. Vectors $\mathbf{u}_1 = (5, 3)$ and $\mathbf{u}_2 = (3, 2)$ form another basis for $\mathbb{R}^2$.

(i) Find the transition matrix from the ordered basis $\mathbf{v}_1, \mathbf{v}_2$ to the standard basis $\mathbf{e}_1, \mathbf{e}_2$ and the transition matrix from $\mathbf{e}_1, \mathbf{e}_2$ to $\mathbf{v}_1, \mathbf{v}_2$.

(ii) Find the transition matrix from the ordered basis $\mathbf{u}_1, \mathbf{u}_2$ to the standard basis $\mathbf{e}_1, \mathbf{e}_2$ and the transition matrix from $\mathbf{e}_1, \mathbf{e}_2$ to $\mathbf{u}_1, \mathbf{u}_2$.

(iii) Find coordinates of the vector $\mathbf{w} = (10, 7)$ relative to the basis $\mathbf{v}_1, \mathbf{v}_2$ and coordinates of $\mathbf{w}$ relative to the basis $\mathbf{u}_1, \mathbf{u}_2$.

Problem 2. Vectors $\mathbf{v}_1 = (4, 6, 7)$, $\mathbf{v}_2 = (0, 1, 1)$ and $\mathbf{v}_3 = (0, 1, 2)$ form a basis for the vector space $\mathbb{R}^3$. Vectors $\mathbf{u}_1 = (1, 1, 1)$, $\mathbf{u}_2 = (1, 2, 2)$ and $\mathbf{u}_3 = (2, 3, 4)$ form another basis for $\mathbb{R}^3$.

(i) Find the transition matrix from the standard basis $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ to the ordered basis $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$.

(ii) Find the transition matrix from the ordered basis $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ to the ordered basis $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$.

(iii) Find coordinates of the vector $\mathbf{w} = 2\mathbf{v}_1 + 3\mathbf{v}_2 - 4\mathbf{v}_3$ relative to the basis $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$, coordinates of $\mathbf{w}$ relative to the basis $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$, and coordinates of $\mathbf{w}$ relative to the standard basis.

Problem 3. Given

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \qquad \mathbf{v}_2 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \qquad U = \begin{pmatrix} 3 & 5 \\ 1 & -2 \end{pmatrix},$$

find vectors $\mathbf{w}_1$ and $\mathbf{w}_2$ so that U will be the transition matrix from the ordered basis $\mathbf{w}_1, \mathbf{w}_2$ for $\mathbb{R}^2$ to the ordered basis $\mathbf{v}_1, \mathbf{v}_2$.

Problem 4. Polynomials $p_1(x) = 1$, $p_2(x) = x$ and $p_3(x) = x^2$ form a basis for the vector space $\mathcal{P}_3$. Polynomials $q_1(x) = 1$, $q_2(x) = 1 + x$ and $q_3(x) = 1 + x + x^2$ form another basis for $\mathcal{P}_3$.

(i) Find the transition matrix from the ordered basis q_1, q_2, q_3 to the ordered basis p_1, p_2, p_3.

(ii) Find the transition matrix from the ordered basis p_1, p_2, p_3 to the ordered basis q_1, q_2, q_3.

(iii) Find coordinates of the polynomial $r(x) = 2x^2 + 3x - 1$ relative to the ordered basis q_1, q_2, q_3.