

## Homework assignment #10

**Problem 1.** Find the distance from the point  $(1, 2)$  to the line  $4x - 3y = 0$ .

**Problem 2.** Let  $V$  be the subspace of  $\mathbb{R}^3$  spanned by  $(1, -1, 1)$ . Find a basis for the orthogonal complement  $V^\perp$ .

**Problem 3.** Find the orthogonal complement of the subspace of  $\mathbb{R}^3$  spanned by vectors  $(1, 2, 1)$  and  $(1, -1, 2)$ .

**Problem 4.** Let  $V$  be the subspace of  $\mathbb{R}^4$  spanned by vectors  $(1, 0, -2, 1)$  and  $(0, 1, 3, -2)$ . Find a basis for the orthogonal complement  $V^\perp$ .

**Problem 5.** If  $A$  is an  $m \times n$  matrix of rank  $r$ , what are the dimensions of the nullspaces  $N(A)$  and  $N(A^T)$ ? Explain.

**Problem 6.** For each of the following systems, find the least squares solution:

$$\begin{array}{lll}
 \text{(i)} \quad \begin{cases} x_1 + 2x_2 = 3 \\ 2x_1 + 4x_2 = 2 \\ -x_1 - 2x_2 = 1 \end{cases} & 
 \text{(ii)} \quad \begin{cases} -x_1 + x_2 = 10 \\ 2x_1 + x_2 = 5 \\ x_1 - 2x_2 = 20 \end{cases} & 
 \text{(iii)} \quad \begin{cases} x_1 + x_2 + x_3 = 4 \\ -x_1 + x_2 + x_3 = 0 \\ -x_2 + x_3 = 1 \\ x_1 + x_3 = 2 \end{cases}
 \end{array}$$

**Problem 7.** Consider the following data:

$$\begin{array}{c|c|c|c|c}
 x & -1 & 0 & 1 & 2 \\
 \hline
 f(x) & 0 & 1 & 3 & 9
 \end{array}$$

(i) Find a linear polynomial which is the best least squares fit to the data. Plot the graph along with the data on a coordinate system.

(ii) Find a quadratic polynomial which is the best least squares fit to the data. Plot the graph along with the data on a coordinate system.