

Homework assignment #9

(due Monday, November 20)

Problem 1. Let $F(\omega)$ be the Fourier transform of $f(x)$. Show that if $f(x)$ is real then $\overline{F(\omega)} = F(-\omega)$.

Problem 2. If $F(\omega) = e^{-\alpha|\omega|}$ ($\alpha > 0$), determine the inverse Fourier transform of $F(\omega)$.

Problem 3. Solve Laplace's equation in an infinite strip

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad (0 < x < L, \quad -\infty < y < \infty)$$

subject to

$$u(0, y) = g_1(y), \quad u(L, y) = g_2(y).$$

Problem 4. Solve a boundary value problem for Laplace's equation in a quarter-plane:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad (0 < x < \infty, \quad 0 < y < \infty),$$

$$u(0, y) = 0 \quad (0 < y < \infty),$$

$$\frac{\partial u}{\partial y}(x, 0) = f(x) \quad (0 < x < \infty).$$

Problem 5. Solve

$$\frac{\partial u}{\partial t} = k \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \quad (-\infty < x < \infty, \quad 0 < y < H)$$

subject to the initial condition

$$u(x, y, 0) = f(x, y)$$

and the boundary conditions

$$u(x, 0, t) = u(x, H, t) = 0.$$