EXERCISES 0

12. Let $A = \{1, 2, 3\}$ and $B = \{2, 4, 6\}$. For each relation between A and B given as a subset of $A \times B$, decide whether it is a function mapping A into B. If it is a function, decide whether it is one to one and whether it is onto B.

a. $\{(1, 4), (2, 4), (3, 6)\}$

b. $\{(1, 4), (2, 6), (3, 4)\}$

c. $\{(1, 6), (1, 2), (1, 4)\}$

d. $\{(2, 2), (1, 6), (3, 4)\}$

e. $\{(1, 6), (2, 6), (3, 6)\}$

f. $\{(1, 2), (2, 6), (2, 4)\}$

19. Show that the power set of a set A, finite or infinite, has too many elements to be able to be put in a one-to-one correspondence with A. Explain why this intuitively means that there are an infinite number of infinite cardinal numbers. [*Hint:* Imagine a one-to-one function ϕ mapping A into $\mathscr{P}(A)$ to be given. Show that ϕ cannot be onto $\mathscr{P}(A)$ by considering, for each $x \in A$, whether $x \in \phi(x)$ and using this idea to define a subset S of A that is not in the range of ϕ.] Is *the set of everything* a logically acceptable concept? Why or why not?

EXERCISES 1

compute the given expression using the indicated modular addition.

27. $2\sqrt{2} +_{\sqrt{32}} 3\sqrt{2}$

find *all* solutions x of the given equation.

33. $x +_{12} x = 2$ in $\mathbb{Z}_{12}$

EXERCISES 2

6. Table 2.28 can be completed to define an associative binary operation $*$ on $S = \{a, b, c, d\}$. Assume this is possible and compute the missing entries.

2.28 Table

$*$	a	b	c	d
a	a	b	c	d
b	b	a	c	d
c	c	d	c	d
d				

In Exercises 7 through 11, determine whether the binary operation $*$ defined is commutative and whether $*$ is associative.

10. $*$ defined on $\mathbb{Z}^+$ by letting $a * b = 2^{ab}$

13. How many different commutative binary operations can be defined on a set of 2 elements? on a set of 3 elements? on a set of n elements?

In Exercises 17 through 22, determine whether the definition of $*$ does give a binary operation on the set. In the event that $*$ is not a binary operation, state whether Condition 1, Condition 2, or both of these conditions on page 24 are violated.

22. On $\mathbb{Z}^+$, define $*$ by letting $a * b = c$, where c is the largest integer less than the product of a and b.

either prove the statement or give a counterexample.

28. Every commutative binary operation on a set having just two elements is associative.

37. Suppose that $*$ is an associative and commutative binary operation on a set S. Show that $H = \{a \in S \mid a * a = a\}$ is closed under $*$. (The elements of H are **idempotents** of the binary operation $*$.)