MATH 415–502 Fall 2022

Homework assignment #4

Problem 1. Find the order and the sign of the following permutations in S_8:

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 5 & 6 & 3 & 7 & 4 & 2 & 1 \end{pmatrix}, \qquad \tau = (1\ 4\ 5)(3\ 8\ 6)(2\ 5\ 7).$$

Problem 2. Find the maximum possible order for a permutation in S_{10}.

Problem 3. Suppose that a permutation σ is a cycle of odd length. Prove that σ^2 is also a cycle.

Problem 4. Prove that any permutation in S_n different from the identity map can be written as a product of at most $n-1$ transpositions.

Problem 5. Suppose H is a subgroup of the symmetric group S_n. Prove that either all permutations in H are even or exactly half of them are even.

Problem 6. Find all cosets of the cyclic subgroup $\langle 3 \rangle$ of the group $\mathbb{Z}_{12}$.

Problem 7. Find all left and right cosets of the cyclic subgroup $\langle (1\ 2) \rangle$ of the group S_3.

Problem 8. Suppose that H is a subgroup of index 2 in a group G. Show that every left coset of H in G is also a right coset of H.

Problem 9. Consider a permutation $\sigma = (1\ 2\ 5)(3\ 4)$ in S_5. Find the index of the cyclic subgroup $\langle \sigma \rangle$ in S_5.

Problem 10. Let G be a group of order pq, where p and q are prime numbers. Prove that every proper subgroup of G is cyclic.