Sample problems for the final exam

Any problem may be altered or replaced by a different one!

Problem 1 (15 pts.) Find a quadratic polynomial $p(x)$ such that $p(-1) = p(3) = 6$ and $p'(2) = p(1)$.

Problem 2 (20 pts.) Consider a linear transformation $L : \mathbb{R}^5 \to \mathbb{R}^2$ given by

$$L(x_1, x_2, x_3, x_4, x_5) = (x_1 + x_3 + x_5,\ 2x_1 - x_2 + x_4).$$

Find a basis for the null-space of L, then extend it to a basis for $\mathbb{R}^5$.

Problem 3 (20 pts.) Let $\mathbf{v}_1 = (1, 1, 1)$, $\mathbf{v}_2 = (1, 1, 0)$, and $\mathbf{v}_3 = (1, 0, 1)$. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear operator on $\mathbb{R}^3$ such that $T(\mathbf{v}_1) = \mathbf{v}_2$, $T(\mathbf{v}_2) = \mathbf{v}_3$, $T(\mathbf{v}_3) = \mathbf{v}_1$. Find the matrix of the operator T relative to the standard basis.

Problem 4 (20 pts.) Let $R : \mathbb{R}^3 \to \mathbb{R}^3$ be the operator of orthogonal reflection in the plane Π spanned by vectors $\mathbf{u}_1 = (1, 0, -1)$ and $\mathbf{u}_2 = (1, -1, 3)$. Find the image of the vector $\mathbf{u} = (2, 3, 4)$ under this operator.

Problem 5 (25 pts.) Consider the vector space W of all polynomials of degree at most 3 in variables x and y with real coefficients. Let D be a linear operator on W given by $D(p) = \dfrac{\partial p}{\partial x}$ for any $p \in W$. Find the Jordan canonical form of the operator D.

Bonus Problem 6 (15 pts.) An upper triangular matrix is called unipotent if all diagonal entries are equal to 1. Prove that the inverse of a unipotent matrix is also unipotent.