MATH 423-200/500 Spring 2012

Sample problems for Test 1

Any problem may be altered or replaced by a different one!

Problem 1 (20 pts.) Let $\mathcal{P}_3$ be the vector space of all polynomials (with real coefficients) of degree at most 3. Determine which of the following subsets of $\mathcal{P}_3$ are subspaces. Briefly explain.

(i) The set S_1 of polynomials $p(x) \in \mathcal{P}_3$ such that $p(0) = 0$.
(ii) The set S_2 of polynomials $p(x) \in \mathcal{P}_3$ such that $p(0) = 0$ and $p(1) = 0$.
(iii) The set S_3 of polynomials $p(x) \in \mathcal{P}_3$ such that $p(0) = 0$ or $p(1) = 0$.
(iv) The set S_4 of polynomials $p(x) \in \mathcal{P}_3$ such that $(p(0))^2 + 2(p(1))^2 + (p(2))^2 = 0$.

Problem 2 (20 pts.) Let V be a subspace of $\mathcal{F}(\mathbb{R})$ spanned by functions e^x and e^{-x}. Let L be a linear operator on V such that

$$\begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}$$

is the matrix of L relative to the basis e^x, e^{-x}. Find the matrix of L relative to the basis $\cosh x = \frac{1}{2}(e^x + e^{-x})$, $\sinh x = \frac{1}{2}(e^x - e^{-x})$.

Problem 3 (25 pts.) Suppose V_1 and V_2 are subspaces of a vector space V such that $\dim V_1 = 5$, $\dim V_2 = 3$, $\dim(V_1 + V_2) = 6$. Find $\dim(V_1 \cap V_2)$. Explain your answer.

Problem 4 (25 pts.) Consider a linear transformation $T : \mathcal{M}_{2,2}(\mathbb{R}) \to \mathcal{M}_{2,3}(\mathbb{R})$ given by

$$T(A) = A \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

for all 2×2 matrices A. Find bases for the range and for the null-space of T.

Bonus Problem 5 (15 pts.) Suppose V_1 and V_2 are real vector spaces of dimension m and n, respectively. Let $B(V_1, V_2)$ denote the subspace of $\mathcal{F}(V_1 \times V_2)$ consisting of bilinear functions (i.e., functions of two variables $x \in V_1$ and $y \in V_2$ that depend linearly on each variable). Prove that $B(V_1, V_2)$ is isomorphic to $\mathcal{M}_{m,n}(\mathbb{R})$.