

**Sample problems for Test 2**

Any problem may be altered or replaced by a different one!

**Problem 1 (20 pts.)** Find a cubic polynomial  $p(x)$  such that  $p(-2) = 0$ ,  $p(-1) = 4$ ,  $p(1) = 0$ , and  $p(2) = 4$ .

**Problem 2 (25 pts.)** Evaluate a determinant

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ c_1 & c_2 & c_3 & c_4 \\ c_1^2 & c_2^2 & c_3^2 & c_4^2 \\ c_1^3 & c_2^3 & c_3^3 & c_4^3 \end{vmatrix}.$$

For which values of parameters  $c_1, c_2, c_3, c_4$  is this determinant equal to zero?

**Problem 3 (20 pts.)** Let  $A = \begin{pmatrix} 1 & 2 & 0 \\ 1 & 1 & 1 \\ 0 & 2 & 1 \end{pmatrix}$ .

- (i) Find all eigenvalues of the matrix  $A$ .
- (ii) For each eigenvalue of  $A$ , find an associated eigenvector.
- (iii) Find all eigenvalues of the matrix  $A^3$ .

**Problem 4 (25 pts.)** Let  $B = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$ . Find a matrix  $C$  such that  $C^2 = B^2$ , but  $C \neq \pm B$ .

**Bonus Problem 5 (15 pts.)** Let  $X$  be a square matrix that can be represented as a block matrix

$$X = \begin{pmatrix} A & C \\ O & B \end{pmatrix},$$

where  $A$  and  $B$  are square matrices and  $O$  is a zero matrix. Prove that  $\det(X) = \det(A) \det(B)$ .