

## Sample problems for Exam 1

Any problem may be altered, removed or replaced by a different one!

**Problem 1.** Find  $\gcd(1106, 350)$ .

**Problem 2.** Find an integer solution of the equation  $45x + 115y = 10$ .

**Problem 3.** Prove by induction that

$$\frac{1}{4} + \frac{1}{16} + \cdots + \frac{1}{4^n} = \frac{1}{3} \left( 1 - \frac{1}{4^n} \right)$$

for every positive integer  $n$ .

**Problem 4.** When the number  $25^7 \cdot 20^{20} \cdot 18^{12}$  is written out, how many zeroes are there at the right-hand end?

**Problem 5.** Find a multiplicative inverse of 29 modulo 41.

**Problem 6.** Which congruence classes modulo 8 are invertible?

**Problem 7.** Find all integers  $x$  such that  $21x \equiv 5 \pmod{31}$ .

**Problem 8.** Solve the system 
$$\begin{cases} y \equiv 4 \pmod{7}, \\ y \equiv 5 \pmod{11}. \end{cases}$$

**Problem 9.** Find the multiplicative order of 7 modulo 36.

**Problem 10.** Determine the last two digits of  $303^{303}$ .

**Problem 11.** How many integers from 1 to 120 are relatively prime with 120?

**Problem 12.** You receive a message that was encrypted using the RSA system with public key  $(33, 7)$ , where 33 is the base and 7 is the exponent. The encrypted message, in two blocks, is  $5/31$ . Find the private key and decrypt the message.