

Sample problems for Exam 1

Any problem may be altered, removed or replaced by a different one!

Problem 1. Find $\gcd(1106, 350)$.

Problem 2. Find an integer solution of the equation $45x + 115y = 10$.

Problem 3. Prove by induction that

$$\frac{1}{4} + \frac{1}{16} + \cdots + \frac{1}{4^n} = \frac{1}{3} \left(1 - \frac{1}{4^n} \right)$$

for every positive integer n .

Problem 4. When the number $25^7 \cdot 20^{20} \cdot 18^{12}$ is written out, how many zeroes are there at the right-hand end?

Problem 5. Is the number 163 prime? Explain how you know.

Problem 6. Which congruence classes modulo 8 are invertible?

Problem 7. Find a multiplicative inverse of 29 modulo 41.

Problem 8. Find all integers x such that $21x \equiv 5 \pmod{31}$.

Problem 9. Solve the system $\begin{cases} y \equiv 4 \pmod{7}, \\ y \equiv 5 \pmod{11}. \end{cases}$

Problem 10. How many integers from 1 to 120 are relatively prime with 120?

Problem 11. Find the multiplicative order of 7 modulo 36.

Problem 12. Determine the last two digits of 303^{303} .

Problem 13. You receive a message that was encrypted using the RSA system with public key $(33, 7)$, where 33 is the base and 7 is the exponent. The encrypted message, in two blocks, is $5/31$. Find the private key and decrypt the message.