Final projects

Presentation and preliminary report:
- Wednesday, April 27
- Friday, April 29
- Monday, May 2
- Tuesday, May 3

Complete report deadline:
- Monday, May 9

Topics for the final project

General theory:

• Find out whether density of periodic points and topological transitivity imply sensitive dependence on initial conditions.

• Topological entropy. Its computation for topological Markov chains and hyperbolic toral automorphisms.

• Square tilings. The Wang tiles.

Topics for the final project

One-dimensional dynamics (theory):

- Prove that the logistic map $F_\mu(x) = \mu x(1-x)$ has an invariant hyperbolic Cantor set for $\mu > 4$.

- The converse of Sharkovskii's theorem.

- The necessary and sufficient condition of chaoticity for subshifts of finite type.

- The Denjoy example.

Topics for the final project

One-dimensional dynamics (numerical):

• Plot the orbit diagram for the logistic map. Find for which values of the parameter the map admits periodic orbits of period 3, 5, 7, 6, 10, all even periods.

• Plot the orbit diagram for the logistic map. Compute the Feigenbaum constant. Verify the Feigenbaum universality.

• Plot the bifurcation diagram for the standard family of maps of the circle.

Topics for the final project

Higher dimensional dynamics (theory):

• The Markov partition and symbolic dynamics for a hyperbolic toral automorphism (work out an example).
• The Plykin attractor.
• The normal form and the Hopf bifurcation (work out an example).
• The Hopf bifurcation for dynamical systems with continuous time.

Higher dimensional dynamics (numerical):

• The Hénon map. The Hénon attractor.
• The Lozi map. The Lozi attractor.

Topics for the final project

Holomorphic dynamics (theory):

• Prove that for a polynomial of degree at least 2, any neutral fixed point with multiplier 1 belongs to the Julia set.

• Prove that the Julia set of any polynomial P with $\deg P \geq 2$ is infinite.

• Prove the Mandelbrot Dichotomy for quadratic polynomials $Q_c(z) = z^2 + c$: the filled Julia set of Q_c is connected if $|Q_c^n(0)| \not\to \infty$ and has infinitely many connected components otherwise.

• Assuming $|c|$ is small enough, prove that the Julia set of Q_c is a simple closed curve.

Topics for the final project

Holomorphic dynamics (numerical):

• Picture the Mandelbrot set. Zoom in the seahorse valley.

• Picture the Mandelbrot set. Zoom in the elephant valley.

• Picture the Mandelbrot set. Find a small Mandelbrot set.

• Picture examples of the Julia sets for the main cardioid and every bulb of periods 2, 3, and 4.