Final project

Presentation and preliminary report:

- Wednesday, April 25
- Friday, April 27
- Monday, April 30
- Tuesday, May 1

Complete report deadline:

- Monday, May 7

Topics for the final project

- topic already taken
- topic still available

General theory:

- Find out whether density of periodic points and topological transitivity imply sensitive dependence on initial conditions.

- Topological entropy. Its computation for topological Markov chains and hyperbolic toral automorphisms.

Topics for the final project

One-dimensional dynamics (theory):

- Prove that the logistic map $F_\mu(x) = \mu x(1-x)$ has an invariant hyperbolic Cantor set for $\mu > 4$.
- The converse of Sharkovskii's theorem.
- The necessary and sufficient condition of chaoticity for subshifts of finite type.
- The Denjoy example.

Topics for the final project

One-dimensional dynamics (numerical):

- Plot the orbit diagram for the logistic map. Find for which values of the parameter the map admits periodic orbits of period 3, 5, 7, 6, 10, all even periods.
- Plot the orbit diagram for the logistic map. Compute the Feigenbaum constant. Verify the Feigenbaum universality.
- Plot the bifurcation diagram for the standard family of maps of the circle.

Topics for the final project

Higher dimensional dynamics (theory):

- The Markov partition and symbolic dynamics for a hyperbolic toral automorphism (work out an example).
- The Plykin attractor.
- The normal form and the Hopf bifurcation (work out an example).
- The Hopf bifurcation for dynamical systems with continuous time.

Higher dimensional dynamics (numerical):

- The Hénon map. The Hénon attractor.
- The Lozi map. The Lozi attractor.

Topics for the final project

Holomorphic dynamics (theory):

- Prove that for a polynomial of degree at least 2, any neutral fixed point with multiplier 1 is a limit of repelling periodic points.
- Prove that any polynomial P with $\deg P \geq 2$ has infinitely many repelling periodic points while only finitely many attracting periodic points.
- Prove the Mandelbrot Dichotomy for quadratic polynomials $Q_c(z) = z^2 + c$: the filled Julia set of Q_c is connected if $|Q_c^n(0)| \not\to \infty$ and has infinitely many connected components otherwise.
- Assuming $|c|$ is small enough, prove that the Julia set of Q_c is a simple closed curve.

Topics for the final project

Holomorphic dynamics (numerical):

- Picture the Mandelbrot set. Zoom in the seahorse valley.
- Picture the Mandelbrot set. Zoom in the elephant valley.
- Picture the Mandelbrot set. Find a small Mandelbrot set.
- Picture examples of the Julia sets for the main cardioid and every bulb of periods 2, 3, and 4.