Homework Assignment #1 Spring 2012 - MATH308

due Jan 30, 2012 at the beginning of class

Topics covered : direction fields; equations $y' = ay + b$, where a and b are constant; separable equations (corresponds to sections 1.1, 1.2, 2.2 in the textbook). *You do not need to use calculator for this assignment.*

1. Assume that the velocity v of the falling object satisfies the following differential equation:

$$v'(t) = 19.6 - \frac{v}{10} \tag{1}$$

   (a) Find a number v_e such that $v(t) \equiv v_e$ is a solution of equation (1) (in other words find the equilibrium solution of (1)).

   (b) Solve the equation (1) with initial condition $v(0) = 0$. What is the limit of this solution when $t \to +\infty$? How this limiting velocity is related to your answer in the item (a)?

   (c) Find the time that must elapse for the object to reach 50% of the limiting velocity found in the item (b).

   (d) How far does the object fall in the time found in the item (c).

2. Given the differential equation:

$$y' = 2 + 3y \tag{2}$$

   (a) Find all equilibrium solutions.

   (b) Sketch a direction field.

   (c) Given any initial condition $y(0) = y_0$ describe the behavior of the corresponding solution when $t \to -\infty$ and the behavior of the corresponding solutions when $t \to +\infty$.

   (d) Solve the equation (2) for each initial value $y(0) = y_0$ and justify your answer in the item (c) analyzing the obtained solution.

3. Solve the following differential equations (find the general solutions):

   (a) $(1 - x^2)^{1/2} y' + xy = 0$ (b) $y' + \dfrac{x \cos x}{y \cos y} = 0.$

4. Given the differential equation:

$$y' = y - y^2 \tag{3}$$

   (a) Find all equilibrium points.

   (b) Sketch a direction field.

   (c) Based on the sketch of the direction field from the item (b) answer the following questions:

      i. Let $y(t)$ be the solution of equation (3) satisfying the initial condition $y(0) = \frac{1}{2}$. Find the limit of $y(t)$ when $t \to +\infty$ and the limit of $y(t)$ when $t \to -\infty$ (for this you do not need to find $y(t)$ explicitly).

      ii. Find all y_0 such that the solution of the equation (3) with the initial condition $y(0) = y_0$ has the same limit at $+\infty$ as the solution from the item (c)i.

      iii. Let $y(t)$ be the solution of equation (3) with $y(0) = -1$. Decide wether $y(t)$ is monotonically decreasing or increasing and find to what value it approaches when t increases (the value might be infinite).

   (d) (*bonus* - 10 points) Find the solution of the equation (3) with $y(0) = -1$ explicitly. Determine the interval in which this solution is defined.