Homework Assignment #10 Fall 2013 - MATH308

due Friday Oct 16 at the beginning of class

Topics covered : *mechanical vibrations (section 3.7) , forced vibrations (section 3.8) use that the gravitational acceleration* $g = 32\frac{ft}{s^2}$.

1. (a) Determine $\omega_0 > 0$, $R > 0$ and $\delta \in [0, 2\pi)$ so as to write the expression $-3\cos 7t + 4\sin 7t$ in the form $R\cos(\omega_0 t - \delta)$; (you can use a calculator to determine an approximate value of δ);

   (b) A mass weighing 16 lb is attached to a 5 ft-long spring. At equilibrium the spring measures 8.2 ft. Assume that there is no damping. If after this the mass is pushed 2 ft down and then set in motion with downward velocity of 4 ft/s, determine the position u of the mass at any time t.

   (c) Find the natural frequency, the period, the amplitude, and the phase of the motion of the spring-mass system of item (b) (you can use calculator to determine the phase).

   (d) Assume that in the case of the spring-mass system of item (b) there is also a damping and we can change the damping constant. What is the critical damping constant?

2. A mass weighing 32 lb stretches a spring $\frac{8}{3}$ ft. The mass is initially released from rest from a point 2 ft below the equilibrium position, and the subsequent motion takes place in a medium that offers a damping force numerically equal to the instantaneous velocity. If the mass is driven by an external force $F(t) = 20\cos(3t)$, then

   (a) Find the equation of motion.

   (b) Determine the steady state solution of this system.