Homework Assignment #14 — Fall 2013 - MATH308

due Friday Nov 8 at the beginning of class

Sections covered 7.1, 7.2, 7.4

1. Let $A = \begin{pmatrix} 7 & 3 \\ 2 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -6 \\ -5 & 3 \end{pmatrix}$. Compute $AB - BA$.

2. Transform the given equation into a system of first order differential equations:

   (a) $u'' + u' + u = e^t \tan t$

   (b) $y^{(3)} + 4y'' - 4ty = 0$

3. Express the given system of linear differential equations in matrix form:

   (a) $\begin{cases} x_1' &= 2x_1 - 7x_3 \\ x_2' &= 2x_2 - 3x_3 \\ x_3' &= x_1 - 15x_2 + x_3 \end{cases}$ (b) $\begin{cases} x' &= \cos t\, x + t^5\, y - \frac{t^7}{7} \\ y' &= -\sin(t^2)\, x - e^t\, y + \frac{t^9}{9} \end{cases}$

4. Determine whether the following solutions of the the system $x'(t) = Ax(t)$ form a fundamental set of its solutions. If they do, give a general solution of the system.

   (a) $x_1 = e^{2013t} \begin{pmatrix} -7 \\ 3 \end{pmatrix}$, $x_2 = e^{2013t} \begin{pmatrix} 14 \\ -6 \end{pmatrix}$;

   (b) $x^1 = \begin{pmatrix} e^{-4t} \\ -2e^{-4t} \\ 3e^{-4t} \end{pmatrix}$, $x^2 = \begin{pmatrix} -2\cos 5t \\ -3\sin 5t \\ \sin 5t \end{pmatrix}$, $x^3 = \begin{pmatrix} -2\sin 5t \\ 3\cos 5t \\ -\cos 5t \end{pmatrix}$,