Homework Assignment #14 **Fall 2013 - MATH308**

due Friday Nov 8 at the beginning of class

Sections covered 7.1, 7.2, 7.4

1. Let $A = \begin{pmatrix} 7 & 3 \\ 2 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -6 \\ -5 & 3 \end{pmatrix}$. Compute $AB - BA$.

$$AB = \begin{pmatrix} 7 & 3 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} 1 & -6 \\ -5 & 3 \end{pmatrix} = \begin{pmatrix} 7-15 & -42+9 \\ 2+5 & -12-3 \end{pmatrix} = \begin{pmatrix} -8 & -33 \\ 7 & -15 \end{pmatrix}$$

$$BA = \begin{pmatrix} 1 & -6 \\ -5 & 3 \end{pmatrix}\begin{pmatrix} 7 & 3 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 7-12 & 3+6 \\ -35+6 & -15-3 \end{pmatrix} = \begin{pmatrix} -5 & 9 \\ -29 & -18 \end{pmatrix}$$

$$AB - BA = \begin{pmatrix} -8 & -33 \\ 7 & -15 \end{pmatrix} - \begin{pmatrix} -5 & 9 \\ -29 & -18 \end{pmatrix} = \begin{pmatrix} -3 & -42 \\ 36 & 3 \end{pmatrix}$$

2. Transform the given equation into a system of first order differential equations:

(a) $u'' + u' + u = e^t \tan t \Rightarrow u'' = -u' - u + e^t \tan t$

Set $\quad x_1 = u \quad\quad\quad \Rightarrow \quad x_1' = u' = x_2 \quad \Rightarrow$

$\quad\quad\quad\; x_2 = u' \quad\quad\quad\quad\;\; x_2' = u'' = -u' - u + e^t \tan t$

$$\Rightarrow \quad \begin{aligned} x_1' &= x_2 \\ x_2' &= -x_2 - x_1 + e^t \tan t \end{aligned}$$

(b) $y^{(3)} + 4y'' - 4ty = 0 \Rightarrow y^{(3)} = 4ty - 4y''$

Set $x_1 = y$

$x_2 = y'$

$x_3 = y''$

$$\Rightarrow \boxed{\begin{aligned} x_1' &= x_2 \\ x_2' &= x_3 \\ x_3' &= 4tx_1 - 4x_3 \end{aligned}}$$

3. Express the given system of linear differential equations in matrix form:

(a) $\begin{cases} x_1' &= 2x_1 - 7x_3 \\ x_2' &= 2x_2 - 3x_3 \\ x_3' &= x_1 - 15x_2 + x_3 \end{cases}$ (b) $\begin{cases} x' &= \cos t\, x + t^5\, y - \frac{t^7}{7} \\ y' &= -\sin(t^2)\, x - e^t\, y + \frac{t^9}{9} \end{cases}$

$x' = P(t)x + G(t)$

$$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} 2 & 0 & -7 \\ 0 & 2 & -3 \\ 1 & -15 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos t & t^5 \\ -\sin(t^2) & -e^t \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -t^7/7 \\ t^9/9 \end{pmatrix}$$

4. Determine whether the following solutions of the the system $x'(t) = Ax(t)$ form a fundamental set of its solutions. If they do, give a general solution of the system.

(a) $x^1 = e^{2013t}\begin{pmatrix} -7 \\ 3 \end{pmatrix}$, $x^2 = e^{2013t}\begin{pmatrix} 14 \\ -6 \end{pmatrix}$

Check Wronskian at some point t_0. For example, $t_0 = 0$

$$W[x^1, x^2](0) = \begin{vmatrix} -7 & 14 \\ 3 & -6 \end{vmatrix} = 42 - 42 = 0$$

$\Rightarrow \{x^1, x^2\}$ is NOT a fundamental set.

(b) $x^1 = \begin{pmatrix} e^{-4t} \\ -2e^{-4t} \\ 3e^{-4t} \end{pmatrix}, \quad x^2 = \begin{pmatrix} -2\cos 5t \\ -3\sin 5t \\ \sin 5t \end{pmatrix}, \quad x^3 = \begin{pmatrix} -2\sin 5t \\ 3\cos 5t \\ -\cos 5t \end{pmatrix}$

$$W[x^1, x^2, x^3](0) = \begin{vmatrix} 1 & -2 & 0 \\ -2 & 0 & 3 \\ 3 & 0 & -1 \end{vmatrix} = 1 \cdot 0 + 2 \cdot (2-9) = -14 \neq 0.$$

The set $\{x^1, x^2, x^3\}$ is fundamental and the general solution

$$x(t) = c_1 \begin{pmatrix} e^{-4t} \\ -2e^{-4t} \\ 3e^{-4t} \end{pmatrix} + c_2 \begin{pmatrix} -2\cos 5t \\ -3\sin 5t \\ \sin 5t \end{pmatrix} + c_3 \begin{pmatrix} -2\sin 5t \\ 3\cos 5t \\ -\cos 5t \end{pmatrix}$$