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$$= \begin{pmatrix} e^{3t} \left(\left(-\frac{4}{3} + \frac{5}{3} \right) \cos 6t - \left(\frac{4}{3} + \frac{5}{3} \right) \sin 6t \right) - \frac{7}{3} e^{-6t} \\ e^{3t} \left(\left(\frac{4}{3} + \frac{5}{3} \right) \cos 6t + \left(-\frac{4}{3} + \frac{5}{3} \right) \sin 6t \right) \\ -\frac{4}{3} e^{3t} \cos 6t - \frac{5}{3} e^{3t} \sin 6t + \frac{7}{3} e^{-6t} \end{pmatrix} = \begin{pmatrix} e^{3t} \left(\frac{1}{3} \cos 6t - 3 \sin 6t \right) - \frac{7}{3} e^{-6t} \\ e^{3t} \left(3 \cos 6t + \frac{1}{3} \sin 6t \right) \\ -\frac{4}{3} e^{3t} \cos 6t - \frac{5}{3} e^{3t} \sin 6t + \frac{7}{3} e^{-6t} \end{pmatrix}$$

c) $x(t) \xrightarrow[t \rightarrow +\infty]{} 0$ if and only if $C_1 = C_2 = 0 \Rightarrow$

$$x(0) = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = C_3 \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -C_3 \\ 0 \\ C_3 \end{pmatrix} \text{ for } C_3 \in \mathbb{R}$$

d) $x(t) \xrightarrow[t \rightarrow -\infty]{} 0$ if and only if $C_3 = 0 \Rightarrow$

$$x(0) = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + C_2 \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} C_1 - C_2 \\ -C_1 - C_2 \\ C_1 \end{pmatrix}$$

for any $C_1, C_2 \in \mathbb{R}$