

Homework Assignment #15

Fall 2013 - MATH308

due Wednesday Nov 13 at the beginning of class

Section covered 7.5

1. Given the following system of linear differential equations:

$$\begin{cases} x_1' &= x_1 + 2x_2 \\ x_2' &= 4x_1 + 3x_2 \end{cases} \quad (1)$$

- (a) Find the general solution of the system (1).
(b) Find the solution of the the system (1) satisfying the initial conditions: $x_1(0) = 3$, $x_2(0) = 0$.
(c) Find all α_1 and α_2 such that if $x(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$ is the solution of of the system (1) with initial condition $x(0) = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$ then $x(t) \rightarrow 0$ as $t \rightarrow \infty$.
(d) Find all β_1 and β_2 such that if $x(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$ is the solution of of the system (1) with initial condition $x(0) = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}$ then $x(t) \rightarrow 0$ as $t \rightarrow -\infty$.

2. Given the following system of linear differential equations:

$$\begin{cases} x_1' &= -x_1 + x_2 \\ x_2' &= x_1 + 2x_2 + x_3 \\ x_3' &= 3x_2 - x_3 \end{cases} \quad (2)$$

- (a) Find the general solution of the system (2).
(b) Find the solution of the the system (2) satisfying the initial condition $\begin{pmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{pmatrix} = \begin{pmatrix} 6 \\ 5 \\ 14 \end{pmatrix}$