Homework Assignment #16 — Fall 2013 - MATH308

due Monday Nov 18 at the beginning of class

Section covered 7.6

1. Given the following system of linear differential equations:

$$\left\{\begin{array}{lcl} x_1' & = & 6x_1 - x_2 \\ x_2' & = & 5x_1 + 2x_2 \end{array}\right. \tag{1}$$

   (a) Find the general solution of the system (1).

   (b) If $x(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$ is a solution of (1), what is the limit of $x(t)$ as $t \to -\infty$. Does this limit depend on initial conditions?

   (c) Find the solution of the system (1) satisfying the initial conditions: $x_1(0) = -3, \quad x_2(0) = 2$.

2. Given the following system of linear differential equations:

$$\left\{\begin{array}{lcl} x_1' & = & 2x_1 + 5x_2 + x_3 \\ x_2' & = & -5x_1 - 6x_2 + 4x_3 \\ x_3' & = & 2x_3 \end{array}\right. \tag{2}$$

   (a) It is known that $\begin{pmatrix} 28 \\ -5 \\ 25 \end{pmatrix} e^{2t}$ is a particular solution of the system and vector $\begin{pmatrix} 4+3i \\ -5 \\ 0 \end{pmatrix}$ is an eigenvector corresponding to the eigenvalue $-2+3i$ of the coefficient matrix. Find the (real) general solution of the system (2).

   (b) Find the solution of the the system (2) satisfying the initial condition $\begin{pmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \\ 25 \end{pmatrix}$

   (c) Find all $\alpha_1, \alpha_2, \alpha_3$ such that if $x(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}$ is the solution of the system (2) with initial condition $x(0) = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix}$ then $x(t) \to 0$ as $t \to +\infty$.