Homework Assignment 2 in Differential Equations, MATH308

due to Feb 6, 2012

Topics covered : *separable equations; linear equations: method of integrating factor; modeling with first order equation; existence and uniqueness of solutions: linear versus non-linear equations (corresponds to sections 2.1, 2.2, 2.3, 2.4 in the textbook).*

1. Consider the differential equation
$$y' = \frac{x}{y}$$
In each of the following three items find the solution of this equation satisfying the given initial condition in explicit form and determine the interval in which the solution is defined, if the initial condition is
(a) $y(2) = 1$ (b) $y(2) = -1$ (c) $y(1) = 2$.

2. Find the general solution of the differential equation
$$ty' + 3y = \cos t, \quad t > 0.$$
and determine how the solutions behave as $t \to +\infty$.

3. (a) Solve the initial value problem
$$y' - 5y = te^{4t}, \quad y(0) = a \tag{1}$$
(b) How do the solutions of (1) behave as t goes to $+\infty$? Show that this behavior depend on the choice of the initial value a and find the value a_0 for which the transition from one type of behavior to another occurs;
(c) Describe the behavior of the solution of (1) corresponding to the initial condition $y(0) = a_0$, where a_0 is as in the previous item.

4. A tank originally contains 100 gal of fresh water. Then water containing $\frac{1}{2}$ lb of salt per gallon is poured into the tank at a rate of 4 gal/min, and the mixture is allowed to leave at the same rate. After 50 min the process is stopped, and fresh water is poured into the tank at a rate of 4 gal/min, with the mixture again leaving at the same rate. Find the amount of salt in the tank at the end of an additional 25 min.

5. Consider the differential equation
$$(9 - t^2)y' + ty = 2t^2.$$
In each of the following three items determine an interval in which the solution with given initial condition is certain to exist if the initial condition is
(a) $y(1) = 5$ (b) $y(4) = -2$ (c) $y(-5) = 0$.

6. (Bonus-20 points) Before attempting this problem look through the problem 30 of section 2.2, p. 49 of the textbook or review the end of your lecture notes from January 27, when we discussed the equation of the type $y' = f(\frac{y}{x})$ (so-called, homogeneous equations; the main idea here is to make the substitution $u(x) = \frac{y(x)}{x}$.). Then find the general solution of the following equations:
(a) $y' = \dfrac{y}{x} + \dfrac{x}{y}$ (b) $y' = \dfrac{x - y}{x + y}$