Homework Assignment #2 Fall 2013 - MATH308

due September 4, 2013 at the beginning of class

Topics covered : separable equations (corresponds to sections 2.2); direction field and qualitative analysis of autonomous equations on the line (corresponds to sections 1.1 and 2.5).

1. Solve the following initial value problem:

$$(xy^2 + x)y'(x) + x^2y - y = 0, \quad y(1) = 1$$

2. Given the differential equation:

$$y' = y^2 + 5y + 6 \tag{1}$$

   (a) Find all equilibrium points.

   (b) Sketch a direction field.

   (c) Based on the sketch of the direction field from the item (b) answer the following questions:

      i. Let $y(t)$ be the solution of equation (1) satisfying the initial condition $y(0) = -\frac{5}{2}$. Find the limit of $y(t)$ when $t \to +\infty$ and the limit of $y(t)$ when $t \to -\infty$ (for this you do not need to find $y(t)$ explicitly).

      ii. Find all y_0 such that the solution of the equation (1) with the initial condition $y(0) = y_0$ has the same limit at $+\infty$ as the solution from the item (c) i.

      iii. Let $y(t)$ be the solution of equation (1) with $y(0) = 0$. Decide wether $y(t)$ is monotonically decreasing or increasing and find to what value it approaches when t increases (the value might be infinite).

   (d) (*bonus* - 10 points) Find the solution of the equation (1) with $y(0) = 0$ explicitly. Determine the interval in which this solution is defined.

3. (Bonus-30 points) Before attempting this problem look through the problem 30 of section 2.2, p. 49 of the textbook or review the end of your lecture notes from August 28/beginning of lecture notes of August 30 , when we discussed the equation of the type $y' = f(\frac{y}{x})$ (so-called, homogeneous equations; the main idea here is to make the substitution $u(x) = \frac{y(x)}{x}$ to obtain a separable equation.). Then find the general solution of the following equations:

   (a) $(y^2 - 2xy)d\,x + x^2\,dy = 0$;

   (b) $xy' = y - xe^{y/x}$.