Homework Assignment #2  Spring 2017 - MATH308, Regular section

due February 2, 2017 at the beginning of class

Topics covered: method of integrating factor (sections 2.1); mixing model (section 2.3), exact equations and integrating factor (section 2.6).

1. (a) Find the general solution of the differential equation

\[(t^2 - 1)y' = ty + 2t(t^2 - 1), \quad |t| > 1.\]

(Hint: Divide both sides of the equation by \(t^2 - 1).\)

(b) Solve the initial value problem

\[xy' + 3y = \cos x, \quad y(\pi) = \frac{2\pi}{3}.\]

2. A tank contains 200 gal of water and 60 oz of salt. Water containing a salt concentration of \((0.2 + 0.4\sin 4t)\) oz/gal flow into the tank at the rate of 10 gal/min, and the mixture in the tank flows out at the same rate. Let \(Q(t)\) be the amount of salt in the tank at time \(t\).

(a) Write the initial value problem for \(Q(t)\), i.e. the differential equation for \(Q(t)\) and the initial condition for \(Q(0);\)

(b) Find \(Q(t)\) at any time moment.

3. Check if the following equation is exact and if yes, solve the given initial-value problem:

\[y\cos 2x e^{xy} - 2(\sin 2x)e^{xy} + 2x = (3 - x\cos 2x)e^{xy}\frac{dy}{dx}, \quad y(0) = 0.\]

(Hint: Move the right-hand side to the left.)

4. Find the value of parameter \(a\) for which the differential equation

\[(x + ye^{2xy})dx + axe^{2xy}dy = 0\]

is exact, and then find the general solution in the case of this value of \(a\).

5. For the differential equation

\[x + e^y + \left(\frac{x^2}{2} + 2xe^y\right)\frac{dy}{dx} = 0\]

find the integrating factor depending on \(y\) only to make it exact and then solve the equation.