

Homework Assignment 5 in Differential Equations, MATH308

due March 7, 2012

Topics covered : *method of variation of parameters; mechanical and electrical vibrations; forced vibration (corresponds to sections 3.6, 3.7, and 3.8 in the textbook); symbolic solutions of ODE's and plot of solutions using MATLAB (commands `dsolve` and `ezplot`) ; use that the gravitational acceleration $g = 32 \frac{ft}{s^2}$*

1. Use the method of variation of parameter to find the general solution of the given differential equation:

(a) $y'' - 3y' - 10y = 4e^{5t}$;

(b) $y'' - 2y' + y = \frac{e^t}{t}$, $t > 0$.

2. (a) Determine ω_0 , R and δ so as to write the expression $\cos 4t - \sqrt{3} \sin 4t$ in the form $R \cos(\omega_0 t - \delta)$;
(b) Write the expression $\cos 5t - \cos 2t$ as a product of two trigonometric functions of different frequencies.

3. A mass weigh 8 lb stretches a spring 6 in.

(a) Assume that there is no damping. If the mass pulled up 3 in and then released with no initial velocity, determine the position u of the mass at any time t . Find the frequency, period, and amplitude of the motion.

(b) Assume that there is damping and we can change the damping constant. What is the critical damping constant?

4. A spring is stretched 3 in by a mass that weighs 2 lb. The mass is attached to a dashpot mechanism that has a damping constant of $2 \frac{lb \cdot s}{ft}$ and is acted on by an external force of $2 \cos 3t$ lb.

(a) Determine the steady state solution of this system;

(b) If the external force is $2 \cos \omega t$ determine the frequency $\omega > 0$ for which the amplitude of the steady state solution is maximal.

5. Consider the initial value problem

$$u'' + 100u = 3 \cos 9t, \quad u(0) = 0, \quad u'(0) = 0$$

(a) Solve this initial value problem without using computer;

(b) Solve the same initial value problem with MatLab using the command `dsolve` (print and attach the program and the result of the computations);

(c) Plot the solution of this initial value problem using MatLab (try to choose a nice interval $[0, T]$ in which to plot the graph so that the effect of amplitude modulation will be seen).